AMS 101 APPLIED MATHEMATICS FUNDAMENTALS

DEPARTMENT OF APPLIED MILITARY SCIENCE
ROYAL MILITARY COLLEGE OF CANADA

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INTRODUCTION

This package has been developed by the staff and professors of the Department of Applied Military Science (AMS) in order to prepare students for core programme courses where they will be required to demonstrate the ability to solve problems using basic mathematical concepts and simple calculations.

The Applied Mathematics course is designed at the junior level for mature students accepted to attend the Land Force Technical Staff Programme (LFTSP) and the Army Technical Warrant Officers Programme (ATWOP) without an engineering or science background. This pre-course study package introduces students to fundamental mathematical and chemistry principles with an emphasis on military applications. It provides the underpinning concepts and principles that will be needed to understand concepts and principles in core technology courses.

The Office of Prime Responsibility (OPR) for this Pre-course study package is the Directing Staff (DS) appointed as Applied Mathematics Course Director by the Director AMS.

FUNDAMENTALS OF MATHEMATICS

SECTION 1 - INTRODUCTION

> *"Philosophy is written in this grand book, the universe which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth."*
>
> Galileo Galilee in *Assayer*

GENERAL

1. Mathematics is the language of science. It is used to describe, other than by words, the fundamental relationships between objects and their behaviour. To understand the science of physics and chemistry, a firm grasp of mathematics is required.

AIM

2. The aim of this précis is to review the essentials of mathematics and the fundamentals of trigonometry and algebra.

SCOPE

3. The scope of this précis includes:

   a. arithmetic fundamentals;

   b. fundamentals of algebra;

   c. working with exponents and radicals;

   d. graphing and systems of linear equations;

   e. basic geometry and trigonometry; and

   f. logarithms.

SECTION 2 - ARITHMETIC FUNDAMENTALS

NATURAL NUMBERS

4. Definition. Natural numbers are used for counting, for example, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, etc. A natural number is said to be divisible by another if the remainder is zero.

5. Prime and Composite Numbers. A prime number is a natural number, other than 1, which is evenly divisible only by itself and 1, i.e., 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, etc. Numbers that are not prime numbers are called composite numbers.

6. Unique Factorization. Any natural number greater than 1 can be written as a unique product of prime numbers, for example, $12 = 2 \times 2 \times 3$.

7. Prime Factorization. To find the prime factors of a natural number, divide the number by consecutive prime numbers starting with 2. If the number is divisible by a prime number, continue to divide the quotient by the same prime number until no longer possible then try to divide by the next higher prime number, i.e., 3, 5, 7, etc. Repeat the process until the quotient is 1.

INTEGERS

8. Definition. Integers are the set of natural numbers, zero, and negative natural numbers, i.e., ...-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, etc.

9. Operations Using Integers. Addition, subtraction and multiplication of integers will result in integers; however, division of two integers will not necessarily result in an integer unless the dividend is evenly divided by the divisor. For example: $3 \div (-4) = 3/4$ or .75 (not an integer).

RATIONAL NUMBERS

10. Definition. Rational numbers include the set of integers plus all the fractional numbers in between that can be represented as a fraction, for example, 4/5, 6/7 or a/b, etc.

11. Irrational Numbers. Some problems in physics result in numbers that cannot be expressed as fractions, i.e., they are irrational. An example includes $\sqrt{2}$ which can be derived from a physical example of right angled triangle where two sides are of length 1 and the hypotenuse, $h = \sqrt{(1^2 + 1^2} = \sqrt{2}$.

REAL NUMBERS

12. Definition. Real numbers include the set of rational numbers and irrational numbers. A real number is any number that can be represented by an infinite decimal. Rational numbers can be represented by repeating decimals, for example, 7/4 = 1.75000000..., 9 = 9.00000000..., 1/7 = 0.142857142857..., 2/3 = 0.66666..., etc.

13. Non-Repeating Decimals. Irrational numbers can be represented by non-repeating decimals, for example, $\pi = 3.1415926536...$

14. Imaginary Numbers. Many problems in physics result in physical relationships that cannot be solved, i.e., $x^2 = -1$. This is impossible because the square of any real number must be positive. To solve these problems the symbol j (or i) is used, i.e., $j = \sqrt{-1}$. Therefore, the square root of a negative number is expressed as the product of j and the square root of a positive number. Numbers written with the imaginary unit “j” are called imaginary numbers.

COMPLEX NUMBERS

15. Definition. Complex numbers include the set of real numbers and imaginary numbers. A complex number is expressed as follows:

a + bj where a and b are real numbers.

The number represented by a is called the real part and bj is the imaginary part.

PROPERTIES OF REAL NUMBERS

16. Commutative Property

a. Addition. The sum of two numbers is the same regardless of the order in which addition is made (subtraction is not commutative), for example:

(1) $5 + 8 = 8 + 5$, or

(2) $a + b = b + a$.

b. Multiplication. The product of two numbers is the same regardless of the order in which multiplication is made (division is not commutative), for example:

(1) $5 \times 8 = 8 \times 5$, or

(2) $a \times b = b \times a$.

17. Associative Property

a. Addition. The sum of three or more numbers is the same regardless of how numbers are grouped (subtraction is not associative), for example:

(1) $5 + (16 + 25) = (5 + 16) + 25$, or

(2) $a + (b + c) = (a + b) + c$.

b. Multiplication. The product of three or more numbers is the same regardless of how the numbers are grouped (division is not associative), for example:

(1) $5 \times (16 \times 25) = (5 \times 16) \times 25$, or

(2) $a \times (b \times c) = (a \times b) \times c$.

18. Distributive Property of Multiplication with Respect to Addition. The product of a number "a", and the sum of "b + c" is the same as the sum of "a × b" and "a × c", for example:

a. $3\,(5 + 4) = (3 \times 5) + (3 \times 4)$; or

b. $3\,(5 + 4) = (5 + 4) + (5 + 4) + (5 + 4)$

$= (5 + 5 + 5) + (4 + 4 + 4)$

$= (3 \times 5) + (3 \times 4)$.

19. Special Properties of Zero and One

a. Identity Elements:

(1) Addition. The sum of any number and zero is the original number. Zero is called

the identity element of addition.

(2) Multiplication. The product of any number and one is the original number. One is called the identity element of multiplication.

b. Using Zero in Multiplication and Division:

(1) The product of any number and zero is zero.

(2) The quotient of zero divided by any number except zero is zero.

(3) The quotient of any number divided by zero is undefined. If $18 \div 0 =$ some number, then some number $\times\ 0 = 18$. This contradicts the rule defined above. However, an examination of the trend:

(a) $1 \div 1 = 1$,

(b) $1 \div .1 = 10$,

(c) $1 \div .01 = 100$,

(d) $1 \div .001 = 1000$,

(e) $1 \div .0001 = 10000$, and

(f) $1 \div .00001 = 100000$, and so on until the divisor equals zero,

indicates that any number divided by zero results in a quotient that is ***infinite***, which is essentially undefined.

20. Inequalities. Numbers or expressions that are not equal are shown using the inequality symbols as follows, $12 > 5$ (12 is greater than 5) or $6 < 8$ (6 is less than 8), or $b > a > c$ (b is greater than a and a is greater than c).

EXPONENTS, ROOTS AND FACTORS

21. Exponential Notation. Exponential notation is used to represent a repeating sequence of multiplication of factors. For example, $2 \times 2 \times 2$ can be represented in exponential form as 2^3 (two raised to the power of three). The number 2 in this example is referred to as the base and the number 3 is referred to as the exponent or power.

22. Squares and Square Roots. Numbers that are raised to the power of two are called squared numbers, i.e., 5^2 means that the number 5 has been squared. The product of 5^2 is $5 \times 5 = 25$. The number 25 is the square of the number 5. The number represented by a is said to be the square root of b if $a^2 = b$, i.e., 5 is the square root of 25. Numbers such as 1, 4, 9, 16, 25, etc., which can be represented in exponential form using a whole number base and an exponent of 2 are called perfect squares. The radical sign, √, is used to show the square root operation, i.e., $\sqrt{25} = 5$. In exponential form, the square root operation is shown as $b^{½} = a$.

23. Additional Roots. The number a is said to be the cube root of b if $a^3 = b$, i.e., 5 is the cube root of 125. Using the radical sign, this is written by placing the number 3 in the crook of the radical sign, i.e., 125 = 5. In exponential form, the square root operation is shown as b– = a. Fourth, fifth, sixth, etc., roots are constructed in the same manner.

24. Multiplication and Division of Exponential Numbers. Provided numbers are of the same base, multiplication and division of exponential numbers is straightforward addition and subtraction of the exponents. For example, 4^5 x $4^6 = (4 \times 4 \times 4 \times 4 \times 4) \times (4 \times 4 \times 4 \times 4 \times 4 \times 4) = 4^{(5+6)} = 4^{11}$. Similarly, $4^5 \div 4^6 = (4 \times 4 \times 4 \times 4 \times 4) \div (4 \times 4 \times 4 \times 4 \times 4 \times 4) = 4^{(5-6)} = 4^{-1} = 1/4$.

ORDER OF OPERATIONS

25. When combining the basic arithmetic operations of addition, subtraction, multiplication, division and exponentiation, the order in which they must be carried out requires a set of rules. The Order of Operation Rules are as follows:

a. evaluate all exponential expressions;

b. multiply or divide, whichever comes first, from left to right; then

c. add or subtract, whichever comes first, from left to right.

FRACTIONS

26. Definition. A fraction can be interpreted in three ways:

a. a fraction is one equal part or several equal parts of one unit, i.e., a slice of pie is a fraction of the whole pie;

b. a fraction is an indicated division, i.e., 3 ÷ 5 = 3/5; or

c. a fraction is a ratio of two numbers, i.e., the ratio of 4 to 9 = 4/9.

27. Terms of Fractions. The numerator (the top number of a fraction) indicates the number of equal parts of one unit while the denominator indicates the number of equal parts the unit has been divided by. The fraction bar separates the two numbers.

28. Fundamental Principles

a. Addition and Subtraction. Fractions must have the same denominator before addition or subtraction can be performed.

b. Multiplication and Division. If the numerator and the denominator are multiplied by the same number, the overall value of the fraction will not change. However, if the numerator or denominator are multiplied by different numbers, the fraction value does change according to the ratio of the two factors. This same property holds true for division operations.

c. Fraction Equivalence and Lowest Terms. A fraction which can be obtained from a given fraction by multiplying both the numerator and the denominator by the same number is said to be equivalent to the given fraction. A fraction is said to be in lowest terms when all the common factors between the numerator and the denominator have been divided out.

d. Lowest Common Denominator. The lowest common denominator of a group of fractions is the smallest number into which all the denominators divide evenly.

29. Addition and Subtraction of Whole Numbers and Fractions. Whole numbers can be represented as fractions by making the number in question the numerator and the number 1 the denominator. Any operations involving whole numbers mixed with fractions can then be evaluated by converting the whole numbers to fractional form and then converting all the fractions to their lowest common denominator form.

30. Multiplication and Division of Whole Numbers and Fractions. A fraction multiplied by a whole number operates only on the numerator of the fraction while division of a fraction by a whole number is a

multiplication of the denominator by the whole number. Similarly, in multiplication of one fraction by another, the numerators of both fractions are multiplied together while the denominators of both fractions are multiplied together. The results are placed in their respective numerator and denominator positions of the resulting fraction. Division of a fraction by another fraction is conducted by inverting the dividend and then multiplying the whole numbers can be represented as fractions by making the number in question the numerator and the number 1 as the denominator. Any operations involving whole numbers mixed with fractions can then be evaluated by converting the whole numbers to fractional form and then converting all the fractions to their lowest common denominator form.

DECIMALS

Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten Thousandths	Hundred Thousandths
1000	**100**	**10**	**1**	**•**	**1/10**	**1/100**	**1/1000**	**1/10000**	**1/100000**

Figure 1-1

Decimals

31. Fractions whose denominators are 10 or powers of 10 such as 100, 1000, etc, are called decimal fractions. Decimal fractions can be represented in decimal form by extending the place value system of whole numbers to the right of the units column. A decimal point is used to separate the units column from the first column of the extended place value system. The diagram above explains the extended place value system.

32. Any fraction can be represented in decimal form by simply multiplying or dividing both its numerator and denominator by a number that will result in a denominator of a power of 10. If this process results in repeating decimal places, then a dot is placed over the repeating digit. If additional calculations are required then the number may be rounded to the required number of digits.

33. Rounding. To round a real number in decimal form, simply identify the decimal place of the number to round to and check the next digit to its right. If this next digit is less than 5 then leave the decimal place in question unchanged and truncate all digits to its right. If the digit to the right is greater than or equal to 5 then increase the decimal place in question by one and then truncate all the digits to its right.

PERCENTS

34. Definition. The percent notation, %, identifies fractions that have been reduced to a denominator of 100. The word percent is derived from the Latin word "per centum" which means "by the hundred". Percents are commonly used as ratios comparing numbers to 100.

35. Percent Equivalents. Numbers can be written in fractional, decimal or percent form, for example, 3% = 3/100 = 0.03.

36. The Three Basic Types of Percent Problems

a. Type 1. Example, 12% of 500 is what number?

$12\% \times 500 = 12/100 \times 500 = (12 \times 500)/100 = 6000/100 = 60$

b. Type 2. Example, 5% of what number is 10?

10 ÷ 5% = 10/(5/100) = 1000/5 = 200

c. Type 3. Example, 25 is what percent of 400?

25 ÷ 400 = 25/400 = 1/16 = 12.5/100 = 6.25%

37. Percent Increase or Percent Decrease

a. Percent Increase. Increase the number B by R percent:

New Amount, N = B + B × R, where R is written in decimal or fractional form.

b. Percent Decrease. Decrease the number B by R percent:

New Amount, N = B - B × R, where R is written in decimal or fractional form.

WORKING WITH SIGNED NUMBERS

38. Introduction. The sign of a number indicates its position relative to zero. Numbers with a negative sign (-) lie to the left (or below) the zero and increase in value the further they are from zero. Numbers with a positive sign (+) or no sign at all lie to the right (or above) zero and increase in value the further they are from zero.

39. Multiplication, Division and Absolute Value. The absolute value (or magnitude) of a number is its positive number with no sign applied, i.e., |-3| = 3 and |3| = 3. An absolute value is indicated by enclosing the number in | |. The magnitude of a number indicates the size of the distance from zero where the number lies. Consequently, -3 is actually +3 but in the opposite direction. Multiplying by -1 causes the number to change direction, -3 = -1 × (+3). Similarly, +3 is actually -3 but in the opposite direction. As a result, +3 = -1 × (-3). Therefore, the rules for multiplication and division of signed numbers are as follows:

a. a positive number multiplied or divided by a positive number results in a positive number;

b. a positive number multiplied or divided by a negative number results in a negative number;

c. a negative number multiplied or divided by a positive number results in a negative number; and

d. a negative number multiplied or divided by a negative number results in a positive number.

40. Addition and Subtraction. Before addition and subtraction of signed numbers can be conducted, they must be resolved into their net effects. For example, (-4) + (-5) must be resolved to (-4) - 5 = -9. Similarly, (4) + (-5) = (4) - 5 = -1 and (4) - (-5) = 4 + 5 = 9.

VARIABLE EXPRESSIONS

41. Introduction. A variable expression is a mathematical equality (or inequality) expression where the actual values of one or more numbers is left unknown or evaluated. For example, if a car travels at a speed of 100 km/hr for a period of 1 hr, it travels a distance of 100 km. This relationship can be represented mathematically as follows:

Let V = the speed of the car

t = the total time driving, and

d = the distance travelled.

Therefore, d = V × t. In this example, V = 100 km/hr and t = 1 hr; therefore, d = 100 × 1 = 100 km.

42. Constants and Variables. A constant is a symbol which represents only one number whereas a variable is a symbol which can represent any value.

43. Algebraic Notation:

a. Multiplication. $8(9) = 8 \cdot 9 = 8 \times 9$ = 8 times 9, and $a(b) = ab = a \cdot b$ = a times b, and abc = a times b times c.

b. Division. $8/(9) = 8/9 = 8 \div 9$ = 8 divided by 9, and $a/(b) = a/b = a \div b$ = a divided by b, and a/b/c = a divided by b divided by c.

c. Exponential Notation. 8^3 = 8 times 8 times 8, and x^4= x times x times x times x, and a^b = a raised to the power b.

SECTION 3 - FUNDAMENTALS OF ALGEBRA

ALGEBRAIC EXPRESSIONS

44. General. Algebraic expressions are variable expressions and are made up of constants, variables, symbols of operations and groupings of symbols. One single element of an expression (consisting of a combination of products and quotients) is referred to as a term. An algebraic expression is made up of one or more terms. For example, $2x^2 + 4xy + y^2 = 0$ is an algebraic expression relating the terms $2x^2$, $4xy$, y^2, and 0.

45. Evaluating Addition and Subtraction. Terms in an algebraic expression cannot be evaluated for addition and subtraction unless they consist of the same "like" terms. For example, $3x^2 + 5x^4$ cannot be evaluated because they are not like terms; however, $3x^2 + 5x^2$ can be evaluated to read $8x^2$. Similarly, $bx + cx$ can be evaluated to read $(b + c)x$.

46. Evaluating Multiplication and Division. Multiplication and division of algebraic expressions cannot be evaluated unless some like terms are available. For example, $(3x^2)(5y^4)$ cannot be completely evaluated because they are not like terms. It can be evaluated to read $15x^2y^4$; however, $(3x^2)(5x^2)$ can be evaluated to read $15x^4$. Similarly, $(bx)(cx)$ can be evaluated to read bcx^2. Fractional expressions identify the quotient aspect of algebraic expressions. For example, $(3x^2)/(5y^4)$ cannot be evaluated further; however, $(3x^2)/(5x^2)$ can be evaluated to read $(3/5)(x^2/x^2)$ which equals 3/5.

47. Fractional Expressions. The rules for addition, subtraction, multiplication and division of numerical fractions applies to algebraic expressions. Common fractions must be reduced (or expanded) to denominators before addition or subtraction can be evaluated; multiplication of a fraction applies to the numerator of a fractional expression while division is simply multiplication of the denominator of a fractional expression.

SOLVING EQUATIONS

48. General Description. An equation is simply an algebraic expression that includes an equality sign (=) and/or an inequality sign (< or >). In these expressions, the "solution" of the equation is the value (or set of values) of the variables in the expression that make the expression true. For example, if $ax + b = 0$, then the solution to this equation would be the values of a, b, and x that would (when applied to this equation) produce a result of 0.

49. Practical Application of Equations. In many problems of business, science and technology equations are used to show the relationship between properties of objects. For example, the average speed of a car is measured by the distance travelled by the car divided by the time taken. This relationship can be represented as $V = d/t$, where V is the speed or velocity of the car, d is the distance travelled and t is the time taken. This equation shows the equality of the term V to the term d/t.

50. Ordering Terms of an Equation. In general, equations are used to solve for the value of an unknown variable given known values for the other variables. Again, using the average speed equation, if V is known to be 10 km/hr and t is known to be 2 hours then the remaining unknown is d. The equation must be reordered to solve for d. In this case, if both sides of the equation are multiplied by t, then the equation becomes $tV = (d/t)t$, which becomes $tV = d$, or simply $d = Vt$. The general rule for reordering equations is that whatever operation is conducted on one side of the equation must also be applied to the other side so that the net equality remains unchanged.

51. Types of Equations. Equations can involve variables and terms raised to any number or power; however, the two most common equation types involve variables raised to the power of one or two. Equations that involve variables to the first power only are called linear equations. Equations that involve relationships of powers of two are called quadratic equations.

SOLVING WORD PROBLEMS

52. Word problems identify equations in practical applications. Generally speaking, equations spoken in practical terms identify mathematical operations as follows:

a. Multiplication - Using the words like “of” or “times” or “quantity”.

b. Division - Using words like “per” or “ratio”.

c. Addition - Using words like “and”.

d. Subtraction - Using words like “less”.

QUADRATIC EQUATIONS

53. Introduction. As stated earlier, quadratic equations contain variables raised to a power of two or less, generally of the form: $ax^2 + bx + c = 0$, where a, b, and c are constants. This is a fairly special occurrence of the quadratic equation that has many applications in science and technology. The solution (or roots) to this type of equation are values of x that make the equation true. To solve this equation, it must be rearranged such that x = something. There are essentially two methods of solving this equation: factoring and using the quadratic formula.

54. Factoring Quadratic Equations. Factoring quadratic equations is based on the assumption that the equation is the result of two binomial (i.e., consisting of two terms) expressions multiplied together, forexample:

$$(x + 3)(x - 2) = x^2 + 3x - 2x - 6 = x^2 + x - 6 = 0.$$

The solutions to this quadratic equation is x + 3 = 0 (i.e., x = -3) or x - 2 = 0 (i.e., x = 2). There will always be two solutions to a quadratic equation. If it is a perfect square, the solutions may be the same.

55. The Quadratic Formula. If the equation can be rearranged into the general form of the quadratic equation, then a general solution formula can be applied as follows:

Equation of the form $ax^2 + bx + c = 0$, where a, b, and c are constants. Two roots of the equation:

$$x = \frac{-b + \sqrt{(b^2 - 4ac}}{2a} \quad \text{and} \quad x = \frac{-b - \sqrt{(b^2 - 4ac}}{2a}$$

SECTION 4 - WORKING WITH EXPONENTS AND RADICALS

ROOTS AND RADICALS

56. As has already been mentioned, roots are solutions to equations. Essentially, finding the root to a number is the reverse of raising a number to a power. A radical is an indicated root of a number or expression and is denoted using the radical symbol, √. Placing a number within the crook of the radical sign indicates which root to evaluate. The number (or expression) under the radical sign is the radicand.

NEGATIVE AND FRACTIONAL EXPONENTS

57. Negative Exponents. Negative exponents indicate the division of 1 by a number. For example, $4^{-1} = 1/4$. Similarly, $a^{-3} = 1/a^3$.

58. Fractional Exponents. Fractional exponents indicate the "root" operation. For example, $4^{1/2} = \sqrt{4}$ or $\sqrt[3]{125} = 125^{1/3}$ or $a^{1/n}$ = the nth root of a.

Section 5 - Graphing and Systems of Linear Equations

THE RECTANGULAR COORDINATE SYSTEM

59. A graph is essentially a pictorial representation of the relationship between two or more variables. The common graph is linear in nature and considers the relationship between just two variables. The Rectangular Graphing System (also known as the Cartesian Coordinate Graphing System) is the most common (all maps with grid references use this system). The Cartesian System represents each point in a relationship by its position horizontally and vertically as shown in the following diagram:

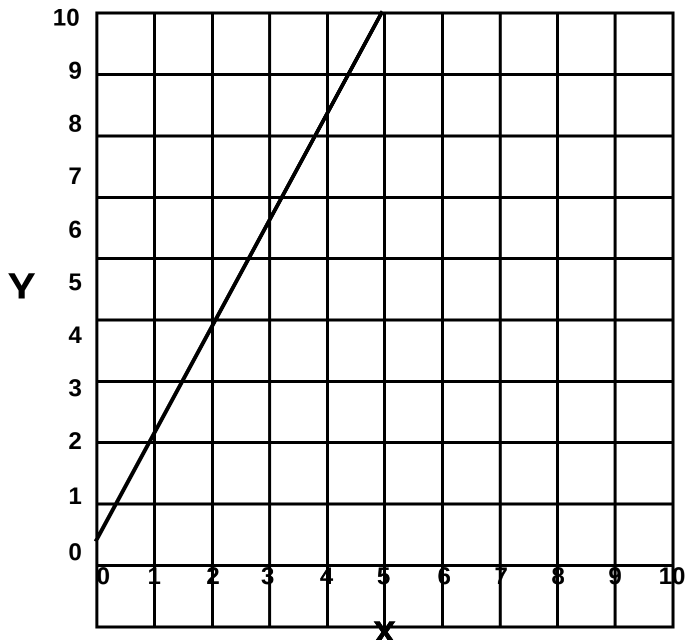


Figure 1-2

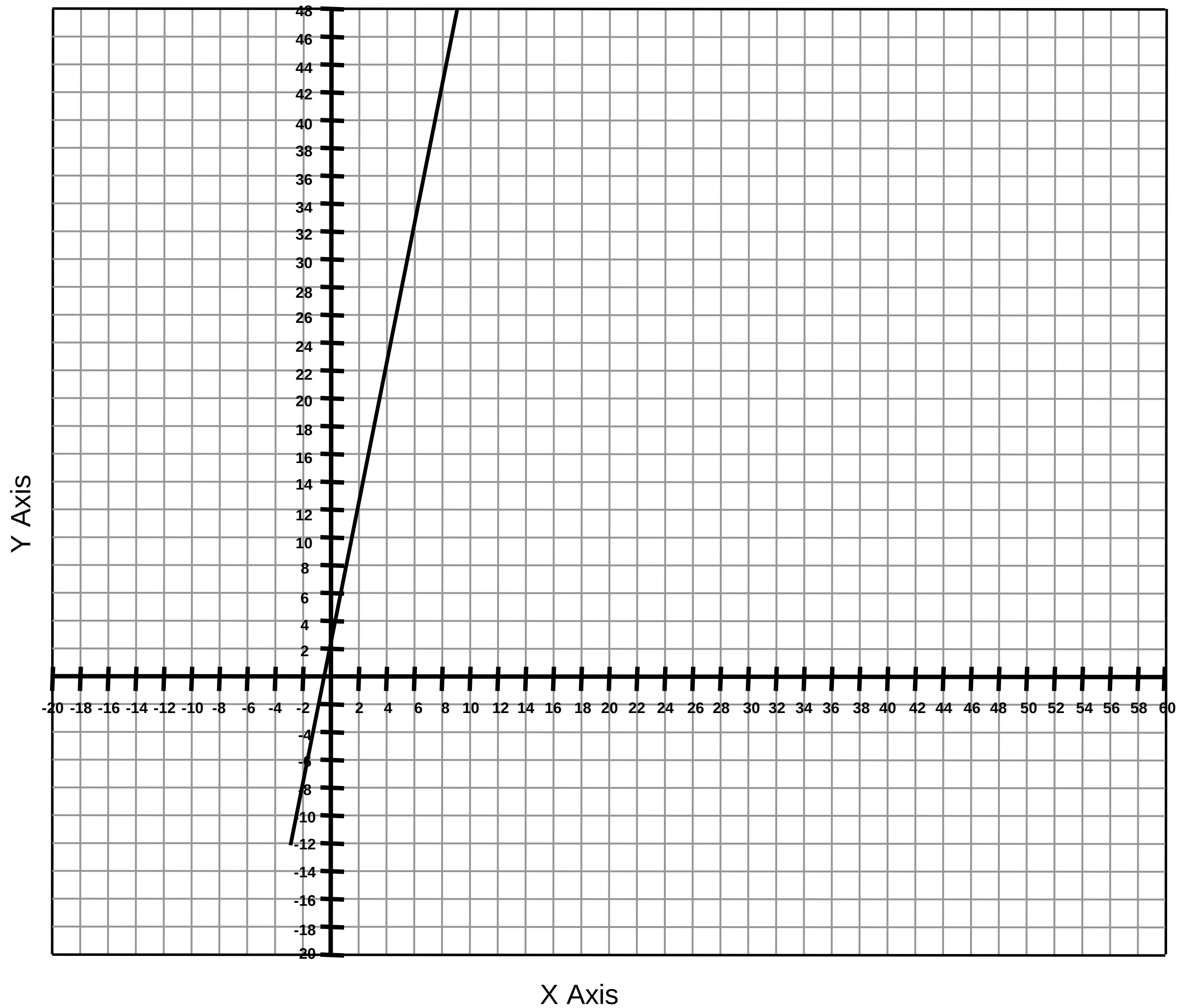
Cartesian Coordinate Graphing System

60. A continuous line between all points in the graph indicates that for every value of X, there is a corresponding value of Y that uniquely satisfies the equation relating the two variables. The most common convention is to use the horizontal axis as the independent variable axis and the vertical axis as the dependent variable axis.

EQUATIONS IN TWO VARIABLES

61. Equations in two variables show mathematically the relationship between the two variables. The graph is used to represent the set of solutions to this equation in a pictorial manner. The graph conveys all the information about the solution to the equation in a way that cannot otherwise be done. The line (or curve) drawn on the graph shows the set of all points (x and y for example) that make the equation true. For example, if $y = 5x + 3$, then the graph of X versus Y would show a line drawn through all the points that satisfy the equation $y = 5x + 3$. To plot this equation on the graph, a table is produced that satisfies the equation. A point is then plotted on the graph for each of these coordinates.

X	-5	-3	-1	0	1	5	10
Y	-22	-12	-2	3	8	28	53



Graph of y = 5x + 3

Figure 1-3

Equations in Two Variables

GRAPHING LINEAR EQUATIONS

62. To graph an equation follow these steps:

a. construct a table of paired values;

b. plot the ordered pairs; and

c. join the plotted points by a smooth curve and label the graph with its equation.

63. The X Intercept. The X intercept is the point at which the equation line crosses the X Axis and occurs when Y = 0.

64. The Y Intercept. The Y intercept is the point at which the equation line crosses the Y Axis and occurs when X = 0.

SYSTEMS OF LINEAR EQUATIONS

65. Linear Equations. Linear equations have the form y = ax + b where a and b are constants. By inspection, the Y intercept = b since x = 0, y = b. The X intercept can be found by rearranging the equation to the form x = (y - b)/a. Therefore, the X intercept = -(b/a) since x = -(b/a) when y = 0.

66. Solution to a System of Linear Equations. A system of equations shows the relationship between two variables under two or more circumstances. The solution to this system of equations is the point (or points) at which the paired x and y coordinates satisfy all equations. When dealing with two variables, this point occurs at the intersection of the two equation graphs. The solution to the system of equations can be solved graphically or using algebra.

67. Solving Simultaneous Equations with Two Unknowns. The substitution method consists of:

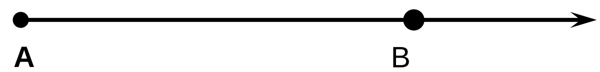
a. solving one equation for one variable in terms of the other variable; and

b. substituting the resulting expression into the other equation.

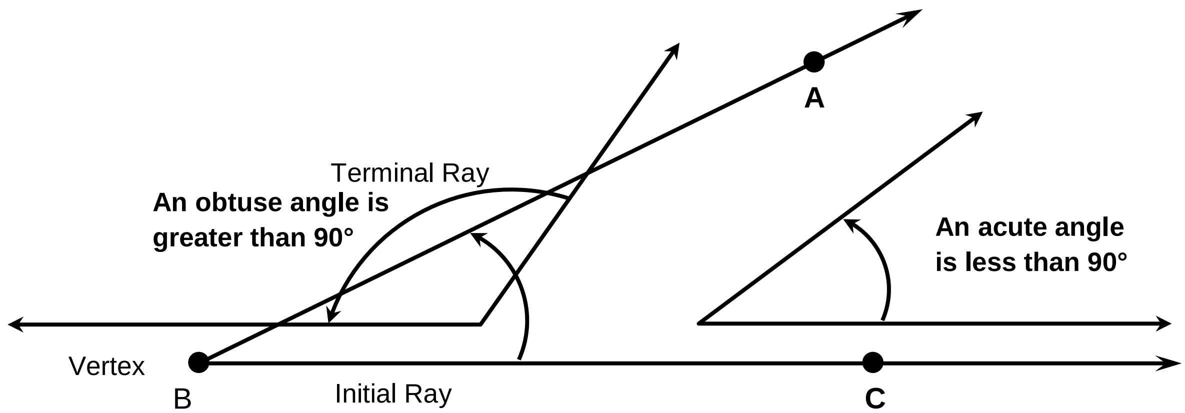
SECTION 6 - BASIC GEOMETRY AND TRIGONOMETRY

BASIC GEOMETRIC CONCEPTS

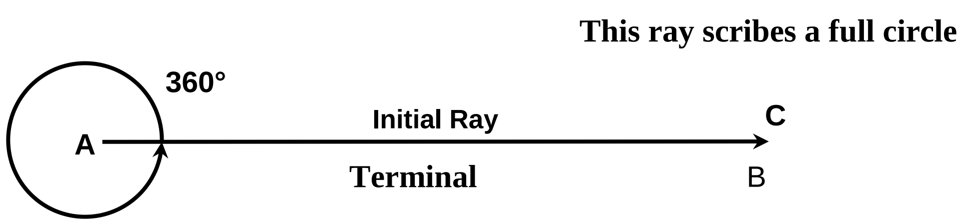
68. Straight Line or Ray. A ray is a line that begins at a definite point but has no end. It is normally named by its beginning point and any other point on it. A line segment is any portion of a line between two points.



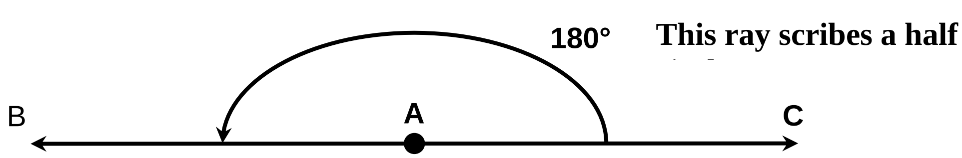
69. Angles. Given two rays with a common beginning point (vertex) an angle is formed by rotating ray BA (the terminal ray) about point B (vertex) from ray BC (the initial ray). The symbol ∠ is used for the word angle. The angle described in the following diagram denotes ∠ABC or simply ∠B.

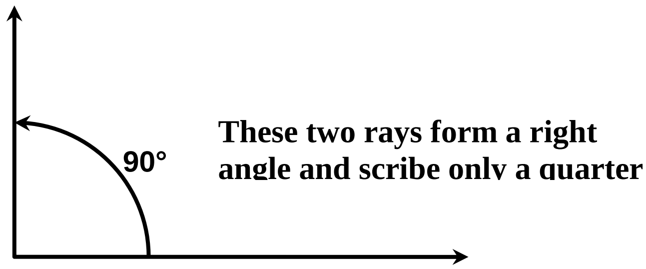


OBLIQUE ANGLES



SPECIAL ANGLES





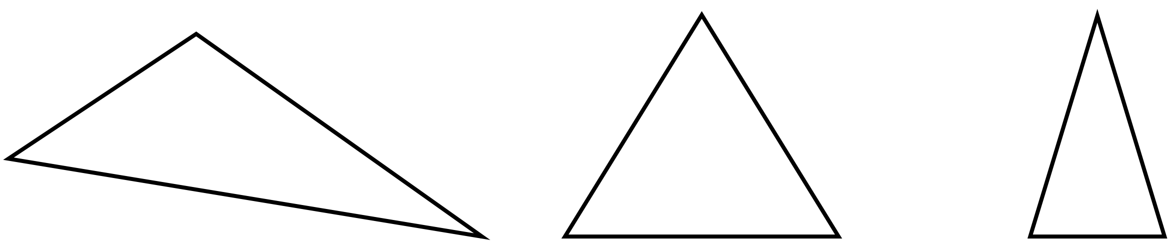
70. Paired Angles

a. Complementary Angles. Two angles are called complementary if their sum is 90^o.

b. Supplementary Angles. Two angles are called supplementary if their sum is 180^o.

TRIANGLES

71. Introduction. A triangle is defined as a plane closed figure formed by three straight lines and having three interior angles. The sum of all the interior angles of a triangle equals 180^o.



A scalene triangle has no equal sides | **An equilateral triangle has all sides equal** | An isosceles triangle has two equal sides

RIGHT-ANGLED TRIANGLES

72. A right-angled triangle is triangle with one angle equal to 90^o. With this fact given, the remainder of the angles within the triangle must add up to 90^o (there are 180^o in the triangle).

73. Trigonometric Properties of a Right-Angled Triangle

a. Pythagorean Theorem $= c^2 = a^2 + b^2$

b. Sine of an Angle = Opposite over Hypotenuse or Sin A = a/c;

c. Cosine of an Angle = Adjacent over Hypotenuse or Cos A = b/c;

d. Tangent of an Angle = Opposite over Adjacent or Tan A = a/b;

e. Cosecant of an Angle = Hypotenuse over Opposite or Csc A = c/a;

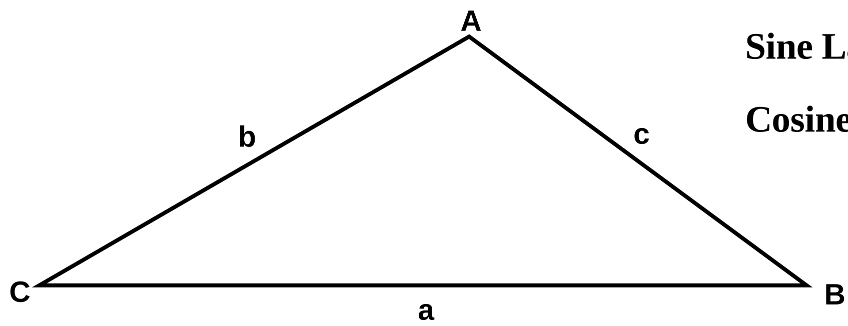
f. Secant of an Angle = Hypotenuse over Adjacent or Sec A = c/b; and

g. Cotangent of an Angle = Adjacent over Opposite or Cot A = b/a.

NOTE: The inverse of these trigonometric functions indicates the angle that satisfies the function result. They are often termed inverse or arc functions.

TRIGONOMETRY OF ANY TRIANGLE

74. Sine Law and Cosine Law

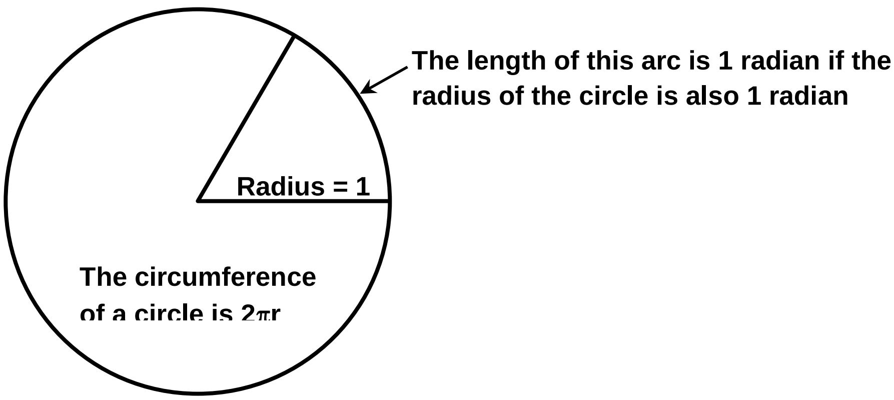


Sine Law: $\frac{a}{\text{Sin A}} = \frac{b}{\text{Sin B}} = \frac{c}{\text{Sin C}}$

Cosine Law: $c^2 = a^2 + b^2 - 2ab \text{ Cos C}$

RADIAN ANGULAR MEASURE

75. Radian angular measure identifies the size of an angle by the length of the arc that angle would draw. As stated earlier, since a circle is drawn by rotating the terminal ray through 360°, the radian measure of this angle is the circumference of the circle if that circle had a radius of 1 radian. The circumference of a circle has been calculated to be 2πr; therefore, the angle of 360° = 2π radians. In other words, 1 radian is the arc length drawn by an angle rotated through an angle of (360°/2π).



SECTION 7 - LOGARITHMS

BASIC CONCEPTS AND LAWS

76. Introduction. A logarithm is another word for an exponent. When working with exponential expressions, there are three general cases of problems:

a. raise a number to a power, for example, $10^3 = x$;

b. find the root (or base) of a number, for example, $x^3 = 27$; and

c. find the exponent (the logarithm function), for example, $10^x = 100$.

77. The Logarithm Function. The logarithm function indicates the exponent of a number and is usually written as $\log_n$. Where n is the base number of the function such that $\log_n a = b$ (logarithmic form) and $n^b = a$ (same question but in exponential form).

78. Special Logarithmic Base Cases. The general notation of the logarithm (or log) function identifies the base used in the function. However, the notation has been simplified when using base ten (the most common) and a special base, e (the natural exponential used in physics):

a. When using Base 10, no subscript base is identified. It is assumed to be base ten unless otherwise noted, log, the common logarithm.

b. When using Base e, the natural logarithm is identified as “ln”.

79. Laws of Logarithms. In general, evaluating expressions involving log functions can only be done if they are evaluated using a common base, for example:

a. $\log_a (mn) = \log_a m + \log_a n$;

b. $\log_a (m/n) = \log_a m - \log_a n$;

c. $\log_a m^k = k\log_a m$; and

d. $\log_a a^k = k$.

EVALUATING EXPONENTIAL AND LOGARITHMIC EXPRESSIONS

80. Finding Common Logarithms. Common logarithms assume that base ten is used. If the number is a simple power of ten, i.e., 1000, then the log function can be found by inspection or counting the number of zeros. In this case log 1000 = 3. For decimal numbers such as 0.0001, the log is found by counting the number of decimal places. In this example, log 0.0001 = -4. If the number is not a simple power of ten, then the best method of finding a logarithm is with the use of a calculator. There are methods of approximating and indeed actually calculating the logarithm function; however, these methods are very advanced are not of any immediate use to most students.

81. Logarithm of any Base Using a Calculator. Most scientific calculators have the common and natural logarithm function already programmed. However, to find the logarithm of a base other than ten or e some additional work must be done, namely, convert the numbers in question to base ten or e as follows:

Solve $\log_3 7 = x$. Rearrange into exponential form as $3^x = 7$

Take common (or natural) log of both sides, resulting in $\log 3^x = \log 7$, which becomes $x\log 3 = \log 7$

Therefore, $x = \log3/\log7$

82. Change of Base. In a change of base problem, we must solve the following type of equation:

$3^x = 7 = 6^y$, change from base 3 to base 6 or more generally, $\log_a k = x$, $a^x = k$,

find y such that $b^y = x$

This is solved much the same as in the previous case to become $\log_a k = (\log_b k)/(\log_b a) = x$