AMS 101

APPLIED MATHEMATICS

PRE-COURSE

STUDY PACKAGE

DEPARTMENT OF APPLIED MILITARY SCIENCE
ROYAL MILITARY COLLEGE OF CANADA

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Contents

INTRODUCTION

This pre-course study package has been developed by the staff and professors of the Department of Applied Military Science (AMS) in order to prepare students for core programme courses where they will be required to demonstrate the ability to solve problems using basic mathematical concepts and simple calculations.

The Applied Mathematics course is designed at the junior level for mature students accepted to attend the Army Technical Staff Officer (ATSO) and the Army Technical Warrant Officers (ATWO) Programme without an engineering or science background. This pre-course study package introduces students to fundamental mathematical and scientific principles with an emphasis on military applications. It provides the underpinning concepts and principles that will be needed to understand concepts and principles in core science and technology courses.

The accompanying AMS 101 Applied Mathematics Fundamentals PDF document provides the theory and process of basic mathematics that will be taught and used throughout the AMS program. However, when learning/reviewing a topic in mathematics, previously it was necessary to consulate a textbook reference, with modern access to the internet, textbook references (for basic mathematics) have become outdated. If the description of a topic presented in this pre-course study package and the AMS 101 Fundamentals is not sufficient, a simple internet search will provide additional sufficient resources at no additional cost. For example if a student needs additional aid in understanding polynomial factoring, simply conduct a google search of "polynomial factoring" and read through resources such as Wikipedia, Khan Academy, or various youtube videos on the subject.

The Office of Prime Responsibility (OPR) for this Pre-course study package is the Directing Staff (DS) appointed as Applied Mathematics Course Director of AMS.

PART I – ADMINISTRATION AND ORIENTATION

CHAPTER 1 – PRE-COURSE STUDY PACKAGE

GENERAL

1. This Chapter of the AMS Pre-course Study package provides an overview of departmental administration and procedures for students to follow to complete the pre-course study package. It will cover:

a. Purpose of the pre-course study package;

b. Study-time;

c. Materials;

d. Study skills and learning outcomes;

e. Study guide;

f. AMS Points of Contact;

g. A self-test; and

h. Self-test Answers.

PURPOSE

2. Background. The staff at AMS recognizes that many students may have been away from the academic forum for quite some time. Consequently, this pre-course study package is designed to assist those students in learning, or re-learning, the fundamentals of mathematics that underpin all subsequent courses in the programme. Mathematics is the language of science and it is essential that the technical and capability management staff can complete basic mathematical problems.

3. Purpose. The purpose of this pre-course study package is to:

a. Assist the individual student in determining which areas he must “brush-up” on;

b. Provide guidance on how to proceed with completing this package; and

c. Guide the student through the materials he must learn.

4. Completion Date. The pre-course study package must be completed prior to the commencement of Introductory Studies on the first day of classes. Students will not have time or opportunity after that date to catch-up, as all subsequent courses are based on the assumption that they have successfully completed this introductory package.

ASSESSMENT

5. Previous Results. Past experience has shown that students who make the diligent effort to complete this study package and practice as required do not have major problems later in the AMS program. Those that do not put in the required effort create a significant burden for themselves and the staff who will provide mentoring. These students will also realize a significant increase in the estimated 17-20 hours of homework/week already expected during the course.

6. Evaluation – Threshold Knowledge Test. All students attending the AMS programmes will be tested prior to course commencement to verify they have completed this pre-courses study package. The test will be based on the learning outcomes found in this package. These results will be used to identify weak students for close monitoring.

LINE OF DEPARTURE

7. Duration. It is estimated that an average student can complete these studies in approximately 30-60 hours.

8. Determine What the Student Already Knows. There are some simple steps to determining what a student knows already. The following steps should help each student:

a. Review the learning outcomes listed in Chapter 6;

b. If the student believes that he/she can pass an evaluation of the lessons, it is recommended that the student attempt the Self Test at Annex B. The student should complete the corresponding section of the self-test and score it themselves, one (1) mark per correct answer;

c. If the score is less than 70%, it is strongly recommended that the student study that topic section and complete all related exercises;

d. As a rough guideline, the mathematics review that will commence upon your arrival at AMS, will begin after the material covered in Annex B: Numbers in Science and Technology and, Week 1 – 2 material outlined in Annex C: Learning Outcomes. The review will quickly go through the remaining chapters before concentrating on the ‘must knows’ for the remainder of the programme. The onus is on you, to make sure you learn the material in this reference and are able to apply it in the remaining courses.

PROVIDED MATERIAL

9. Applied Mathematics Fundamentals. A more detailed description of mathematical theory and processes can be found in the accompanying AMS 101 Applied Mathematics Fundamentals.

10. Self-Test. A self-test exercise is enclosed as Annex B. It is designed to assist the student in determining what he/she already knows. The answers are attached as Appendix 1.

11. Learning Outcomes. The learning outcomes are attached as Annex C. These provide the expected skills that students are expected to achieve at the completion of each section and have prior to arriving at AMS.

12. Learning Schedule. Annex C is organized into a timetable which details;

a. A fourteen week schedule for completing this package;

b. The chapters and topics from the textbook that will be covered;

c. The learning outcomes, a list of what the student must be able to do when they have completed each lesson;

d. A list of the questions from the test at the end of each chapter that you should be able to answer once you have studied that chapter; and

e. Answers to exercises are worked out directly in the Problem Solvers text. You must be able to follow the solution and be able to repeat the process in subsequent questions.

STUDYING AND STUDY SKILLS

14. Studying. Studying is a learned skill that must be practiced. Students will be introduced to basic skills during the programme but development of these skills is an individual student responsibility. Students will be given tools upon arrival at AMS to assist them with study techniques, in the interim students should practice the following:

a. Selecting an appropriate location.

b. Study techniques and ideas, in particular:

(1) Reviewing the learning outcomes then deciding whether to study the lesson on a particular topic or to take the self-test and reviewing the results;

(2) Removing distracting material that may be more interesting than the subject material being studied;

(3) Dealing with urgent, but not study related, issues; and

(4) Getting help when running into difficulty;

(5) Learning will be more effective if it is done over a longer period of time. Don't wait until the last minute to start.

AMS ADVISORS

11. Academic Advisor. The instructor for the Applied Mathematics course will be made available to you during the first two weeks on the course, the advisors' name, phone number and email address will be provided and they will be responsive to your questions within 24hrs (usually within an hour is the expected response time).

12. AMS Point of Contact. The OPI for the Mathematics Pre-Course study is the AMS Infantry Directing Staff, Maj Nate Malazdrewicz. E-Mail: nate.malazdrewicz@rmc-cmr.ca: 613-541-6000 ext 3901.

CHAPTER 2- NUMBERS IN SCIENCE AND TECHNOLOGY

STUDENT PRE-COURSE STUDY PACKAGE

GENERAL

1. The language of military technology is mathematics. It provides the ‘common operating picture’ for military managers, scientists and users to allow them to communicate performance and operational capabilities and requirements between themselves. Without mathematics, operational capabilities cannot be expressed; requirements cannot be translated into technical performance parameters or metrics; and demonstration of actual performance and capabilities by the product cannot be measured. Numbers are also very useful in expressing, understanding and applying concepts when advising commanders and staffs. It is essential that technical and capability management staff understand scientific notations and measurements and can manipulate numbers so that factual decisions can be made based on the results of analysis of numbers.

2. This pre-course study package is intended to provide the basics of all subsequent introductory material and core technologies delivered at AMS. It complements the Math made Simple text and the mathematics course.

DECIMAL FRACTIONS

3. The Decimal System. The numbering system used in the modern world, particularly in science and technology, is called the *decimal system*, which is based on the number 10. The value of any particular digit is determined by its position in a number. For example, in the number 1234, the digit 1 has a value of 1000 or 1 x 1000; the digit 2 has a value of 200 or 2 x 100; the digit 3 has a value of 30 or 3 x 10; and the digit 4 has a value of 4 or 4 x 1.

Decimal fractions. This numbering system can be extended so that the digits represent tenths, hundredths, thousandths and so on. Digits after the decimal point are called decimal fractions of simply a decimal. For example, in the number 1234.567, the digit 5 has a value of 5 tenths or 5 x 1/10; the digit 6 has a value of 6 hundredths or 6 x 1/100; and the digit seven has a value of 7 thousandths or 7 x 1/1000.
Calculations are simplified greatly by the use of decimal fractions. In most military technology applications, the metric system is used, which is a decimal system.

UNITS

4. Measurement. Measuring a physical quantity or characteristic of something is simply the process of comparing it in size (magnitude) to a standard quantity called a *unit*. For example, the mass of a clip of 5 x 25mm ammunition rounds weighs 1.2 kilograms. This means that a kilogram was used as the unit of mass and that the clip weighed 1.2 times the mass of the standard kilogram. The measure for the rounds' diameter was a millimetre, indicating that the round was 25 times the standard millimetre.

> A **measurement** is the ratio of the magnitude of any physical quantity to that of a known standard.

5. Standard Units. A measure of a physical quantity with which other units are compared is called a *standard unit*. All standard units are defined by some legal authority. basic quantities used in technology and science are: *length (L), mass (M), time (T), temperature (°C),* and *electric current (Q or I)* (which will be covered in the physics modules). Whenever we measure something, we have to specify what units we are measuring it in. These units are sometimes referred to as the dimensions. *Dimensions refer to the physical characteristic or quantity of an entity*.

6. SI Units. The standard system of units for science and technology are called *SI* or (System International). Its units of length, mass, time and temperature are the meter, kilogram, second, and degrees Kelvin. It is based on the metric system. The US customary system for units of length, mass and time are the: foot (ft), slug, and second (sec). These may be still encountered in some instances during the courses or field study visits. The table below outlines the common prefixes, power and abbreviations used in AMS courses.

7. Prefixes and Powers for SI units.

Power	Prefix	Abbreviation
10^{-18}	atto-	a
10^{-15}	femto-	f
10^{-12}	pico-	p
10^{-9}	nano-	n
10^{-6}	micro-	µ
10^{-3}	milli-	m
10^{-2}	cent-	c
10^{-1}	deci-	d
10^{1}	deka	da
10^{3}	kilo-	k
10^{6}	mega-	M
10^{9}	giga-	G
10^{12}	tera-	T
10^{15}	peta	P
10^{18}	exa-	E

Table 1A-1. Prefixes and Powers for SI (metric) Units

PRECISION AND ACCURACY

8. Any numerical quantity or characteristic is presented in one of two ways: either as an *exact number*, or as the result of some *measurement*. For example, if you count the number of rounds of 5.56mm ammunition in a magazine and reach 20, 20 is an *exact number* since you can't have a partial round. However, some numerical quantities are the result of a measurement. If we state that the caliber of the ammunition is 5.56mm, that number is the result of some *measurement*.

Example. Let's say the quality control section in the manufacturer's plant examines six (6) measurements of a 5.56mm round's diameter using two teams as it moves through the production process. The teams use the same test instrument at each station, which has been carefully calibrated and handled. The results are:

Measurement #	**Result**
Tm A 1	5.557mm
Tm A 2	5.557mm
Tm A 3	5.557mm
Tm B 4	5.563mm
Tm B 5	5.557mm
Tm B 6	5.560mm

At first glance, it looks like only one measurement station was correct. Assuming that a check of the machine shows that it is still calibrated, how do we deal with, or explain these results? The results are the results of a measurement and are not exact. One thing to do with such repetitive measurements is to average the results by summing the individual measurement results and dividing by the number of measurements made. 5.557 + 5.557 + 5.557 ÷ 3 = 5.56mm (Tm A) and 5.563 + 5.557 + 5.560 ÷ 3 = 5.56mm (Tm B).

In this case, Team A's measurements were more precise because Station 1, 2 and 3 measurements were the same and thus as close to one another as possible. But only team B's measurement 6 was accurate. Team B's average was also more accurate since it matched the 'true' diameter of the round.

9. These results highlight two important aspects of measured numerical quantities or characteristics: *accuracy* and *precision*. Accuracy refers to how close the measured result is to the true value, and precision refers to the closeness to one another of a series of measurements made on the same object. This example shows that precision and accuracy do not go together but of course, the goal in any weapons system is to achieve both precision and accuracy.

Accuracy refers to the degree of agreement between a measured or experimental value and its 'true' or correct value.

Precision refers to the consistency or reproducibility of a measurement

11. High precision does not of itself guarantee high accuracy. For example, in the diagrams below, the left target shows precision; the centre target shows accuracy; and the right target shows precision and accuracy.

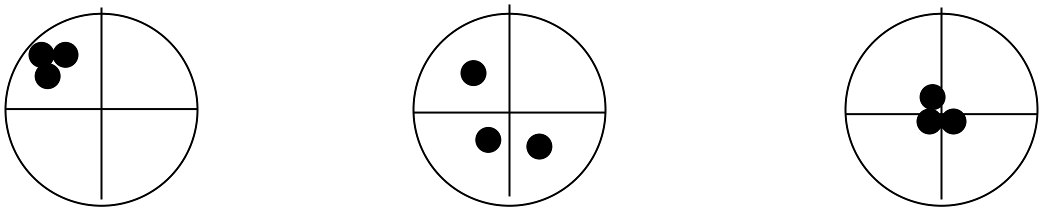


Figure 1A1 – Precision and Accuracy

Practical Exercise

1. Four AMS students were given an essay to write a synopsis of. The essay was 2800 words long but the students didn't know this and counted the number of words to gain an estimate of how long their synopses should be. These were the results:

Fumbles: 2736 words, Rumbles: 2792 words, Bumbles: 2810, Stumbles: 2734

a. Which student was most accurate?
b. Which student was most precise?
c. Is the group's average word count accurate or inaccurate?
d. Four other students did the same thing and came up with word counts of 2722, 2724, 2719, and 2723. Is this group more or less accurate than the first? Is this group more or less precise than the first?

2. Two soldiers count the number of loose 7.62mm rounds in a standard ammunition box. The actual number of rds is 260. They both count the number of rds four times with the following results:

Jones: 256, 263, 262, 266 Smith: 250, 242, 270, 278

a. Which soldier is more accurate?
b. Which is more precise?

UNCERTAINTY

12. No measurement of a physical quantity or characteristic is absolutely exact because there is no way to prove a measured value is absolutely true or correct. In applied technology, no measurement is ever considered to be exact because there is no way to prove that a value is absolutely true or correct. Therefore measurements are considered to be only an approximation to the true value of a measured quantity or characteristic. This creates inherent uncertainties in experiments, tests, or measurements called *experimental errors* or *uncertainties*. There are two types of uncertainties or errors: *systematic* and *random*.

13. Systematic Errors. These are also called constant errors have the same algebraic sign and make measurement of a physical quantity or characteristic either too large or too small. Clocks which gain or lose time are examples of faulty measuring devices which may create systematic uncertainties. Because these are constant, they can be 'corrected' by adjustments to the measuring device or characteristic itself. Barrel wear, or a sight that is not zeroed on a rifle are other examples of a systematic uncertainty that affects accuracy. The systematic errors can be minimized by 'aiming-off' or adjusting the sights so that the error is 'taken-up'.

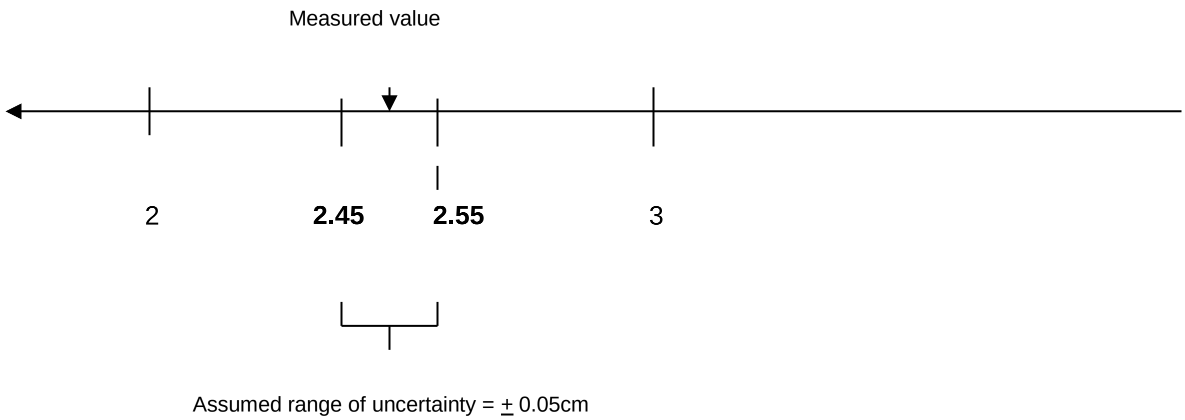
> **Systematic errors** are constant errors that have the same algebraic sign

14. Random Errors. Random errors in measurements of physical quantities result from chance variations in it or the measuring devices. Random errors are generally small and have an equal probability of being positive and negative. They are unavoidable. Typical examples include changes in pressure or temperature, either by small irregularities in the object or characteristic being measured or the measuring device. Random errors are minimized by taking a large number of observations (or measurements) under the same conditions and averaging the measurements.

> **Random errors** are unavoidable errors that result from chance variations

15. Recognizing Uncertainty. Uncertainty is expressed in several ways, the ones you are most familiar with are when seeing measurements expressed as: 16.3 $\pm$ 0.1 or when a measurement is expressed with conditions such as ‘in a vacuum at ^{o}K’ .

16. Uncertainty Rule. The standard convention is to assume *that uncertainty lies with the last digit written in a number*. For example, the measurement of the thickness of a plate of armour, using a ruler, is reported to be 2.5 cm:



The measurement is assumed to be uncertain in the tenths place (where the 5 in the 2.5 number is) and can be written as 2.5 $\pm$ 0.05 cm. This means that the actual number could be anywhere from 2.45 cm to 2.55 cm. The $\pm$ 0.05 is arrived at by placing a 1 in the position of uncertainty (the last digit, in this case the tenths place where the 5 was) and dividing by 2 (0.1 ÷ 2 = 0.05).

Uncertainty lies with the last digit written in a number

Practical Exercise

3. How is uncertainty in a measurement determined?

4. What is the uncertainty in each of the following numbers:

   a. 14.50 mm?

   b. 14.5 mm?

   c. 0.000 000 03 in? ***[1]**

   d. 125 mm?

5. What is the uncertainty in the actual diameter of the following munitions:

   a. 5.56 mm?
   b. 155 mm?
   c. .50 cal?

***[1] Note:** To ease reading and understanding, examples used in the examples and practical exercises of this pre-course study package will leave a space between every three digits, when more than four digits are present. This is a convention just for <u>this pre-course study package only</u>.

SIGNIFICANT FIGURES AND ZEROS

17. Significant Figures. A measurement has high precision only if the random errors are small. One measure of precision is indicated by the use of *significant figures*. Significant figures are the digits measured which not beyond the accuracy of the measuring device used to measure an object. In the example above, if you are told that the plate of armour is 2.5 cm thick, you must assume the uncertainty is $\pm$ 0.05 cm and that all of the digits (2.5) are significant (there are two significant numbers, 2 and .5). As another example, if you are told that a gun tube is 105mm in diameter, the numbers 1, 0 and 5 are significant.

A **significant figure** is a reliably known digit

18. Leading Zeros. What if the thickness was reported as 0.05 cm? Are all three numbers, ie, the 2 x 0's and 5 significant? ***NO***. The only significant figure is the 5. The 0's are just placeholders called *leading zeros*. They are ***not*** significant. Leading zeros are usually found in numbers whose absolute value is smaller than one (1).

Leading zeros are all the zeros that precede the first nonzero digit digit.

19. Uncertainty and Leading Zeros. The rule for where uncertainty lies with measurement of the 0.05 cm armour plate is the same as stated in para 18 above. The last written digit in our plate measurement of 0.05 cm is 5, which is in the hundredths place. Therefore, the uncertainty in the measurement is $\pm$ .005 (place a 1 in the position of uncertainty where the 5 was, and divide by 2, ie, 0.01 ÷ 2 = .005). The plate's thickness should really be written as 0.05 $\pm$ .005 cm.

Leading zeros are not significant

20. Trailing Zeros. Leading zeros come before the last non-zero digit, never after it. Consider the two measurements:

$\underline{0.00}1\ 05$ m & $\underline{0.00}1\ 050$ m

Both of these numbers have 3 x leading zeros, which are underlined. The first number therefore only has 3 significant figures, while the second has 4 significant figures. The last 0 in the second number is a trailing zero.

Trailing zeros are those zeros that appear to the right of the last nonzero digit in a number

21. Trailing zeros that *follow the decimal point are always significant*. In the second number above, the zero is in the millionths place, so the number 0.001 050 is known to an uncertainty of ± 5 ten-millionths (±000 000 5).

Trailing zeros that follow the decimal point are significant

Practical Exercise

6. How many significant figures are in the following numbers:

a. 81.1 ± 0.2 ?

b. 4.688×10^{9} ?

c. 5.56×10^{-6} ?

d. 0.003 2 ?

7. How many significant figures are there in the following measured quantities:

a. 43.20 cm?

b. 0.000 5 m

c. 0.001 117 kg?

d. 1.6×10^{11} c/kg?

e. 6.050 7g?

OPERATIONS WITH SIGNIFICANT NUMBERS

22. Rules for Significant Numbers. The following rules should help when dealing with significant numbers:

a. *The first significant figure in a number is the first digit, other than a zero, counting from left to right.* In the measure 7.62mm, the first significant figure is 7.

b. *Zeros to the left of the first nonzero digit are not significant.* In the measure, 0.002 335 g/cm^3, the zeros to the left of the 2 are not significant. There are 4 significant figures in the number.

c. *Zeros which occur between two significant digits are significant since they are part of the measure*. In the number 30.080 5, the zeros are significant because they occur between the 3 and the 5. There are 6 significant figures in this measure.

d. *Final zeros in measurements containing decimal fractions are significant.* Thus in the measurement 9.0 sec, the final zero is significant since it indicates that the measurement was precise to one-tenth of a second. There are two significant figures.

e. *The number of significant figures is independent of the measurement unit*. In the measurement of our 105mm tank gun, the number could be expressed as 10.5 cm, 105mm, 0.105 m, or 0.000 105 km. In each case, the number of significant figures is 3.

23. Rounding Off Decimal Numbers. In many cases, it is desirable to express a decimal number to fewer places. This process is called *rounding off*. The rules are:

a. *If the first digit to be dropped is less than (<) 5, then the last retained digit is unchanged*;

Example. Round off 3.141 6 to two decimal places. Since 4 is the last digit to be retained, and the next digit to its right is < 5, the rounded number becomes **3.14**.

b. *If the first digit to be dropped is greater than (>) 5, then the last retained digit is increased by 1*;

Example. Round off 0.453 6 to three decimal places. Since 6 is the digit to be dropped and it is > 5, the last retained digit is changed from 3 to 4, the rounded number becomes **0.454**.

c. *If the first digit to be dropped is 5 followed by zeros, then the last retained digit is kept unchanged if it is even. If the last digit is odd, then it is increased by 1*;

Example. Round off 16.450 to one decimal place. The first digit to be dropped is 5 followed by a 0. Since the last digit to be kept is even, 4, it is left unchanged, the rounded number becomes **16.4**.

Example. Round off 193.735 00 to two decimal places. The first digit to be dropped is 5 followed by two 0's. Since the last digit to be kept is odd, 3, which is increased to 4, the rounded number becomes **193.74**.

d. *If the first digit to be dropped is 5 followed by a nonzero digit, then the last retained digit is increased by 1*;

Example. Round off 2.853 0 to one decimal place. The last digit to be kept is 8, it is increased to 9, the rounded number becomes **2.9**.

24. Rounding Off Significant Numbers.

a. *If the first digit to the right of the last significant digit is less than (<) 5, it is dropped*;

b. *If the first digit to the right of the last significant digit is more than (>) 5, then the last significant figure is raised by one and retained*;

c. *If the first digit to be dropped is 5 followed by 0's, then rule a. in para 23 above applies*.

Example. 42.54 ml to 3 significant figures is 42.5 ml, 42.5 ml to 2 significant figures is 42 ml, and 425.6 ml to 3 significant figures is 426.

25. Addition & Subtraction of Significant Numbers. *When significant numbers are added or subtracted, the number of decimal places in the result should equal the smallest number of decimal places in the least precise number*. This means that they should be rounded off to the precision of the least precise measurement.

Example. An APFSDS round's speed of 1331.46 m/s is increased by 514.9 m/s. What is the resultant speed? Round off 1331.46 to the nearest tenth to match the lower precision of the second number. The result is 1331.5 + 514.9 = **1846.4 m/s**.

Example. Add 123 cm, 1.006 cm and 2.6 cm.

  123
\+ 1.006
\+ 2.6

126.606, but rounded off to 127 cm because the least precise or certain of the values (123) is only to that level of uncertainty.

26. Multiplication and Division of Significant Numbers. *When significant numbers are multiplied or divided, the number of significant figures in the result is no greater than the number of significant figures in the measurement with the fewest significant digits least precise measure*.

Example. The dimensions of a reactive armour plate are length = 13.2 cm, width = 4.8 cm. What is the area of the plate? Multiply 13.2 x 4.8 cm = 63.36. cm^2. The answer must be rounded off to 63 cm^2 since 4.8 has only two significant figures.

Example. Multiply 2.0 cm x 2 cm and multiply 2.0 cm x 2.0 cm

2.0 cm	(2 sig figs)	2.0 cm	(2 sig figs)
X 2 cm	(1 sig fig)	X 2.0 cm	(2 sig figs)
4 cm^2	(1 sig fig)	**4.0 cm^2**	(2 sig figs)

27. Pure Numbers and Defined Ratios. A natural number like 6 can be written as 6.0, 6.00, 6.000 and any number of arbitrary zeros. *Multiplication and division of measurements by natural numbers preserves the number of significant figures*. For example, if the measured radius (r) of a circle is 1.96 cm, the diameter of the circle (2r) is 2 x 1.96 cm = 3.92 cm. The number of significant figures remains three because 1.96 has htree significant figures. The same principle applies to quantities whose relationship is determined by definition, conversions result in as many significant figures as there are in the measured quantity. For example, the standard conversion formulae of inches to cms is 1 in = 2.5400 cm. Therefore 3 in = 2.5400 x 3 = 7.6200 cm while 4.3 cm = 4.3 ÷ 2.5400 = 1.7 (1.69291 rounded to the nearest significant digit in 4.3), because it was the measured quantity.

Practical Exercise

8. Subtract 0.0926 from 4.575.

9. The SSM measures the distance around a rectangular field to use as a helicopter landing zone for resupply. The length along each long side of the rectangle is 38.44 m and length along each short side is 19.5 m. What is the total distance around the field?

SCIENTIFIC NOTATION

28. A number is said to be in *scientific notation* if it is expressed as the product of a number between 1 and 10 and some integral power of 10. For example, 1000 written in scientific notation format is 1.0 x 10^3 or simply 10^3; 0.0001 is 10^{-4}; 1247 = 1.247 x 10^3; and 0.000 186 = 1.86 x 10^{-4}. This scientific notation has become the accepted way of indicating uncertainty in a measured number. The general form of scientific notation is therefore:

a. *There should always be a decimal point in the **A** part of the number because this gives you your number of significant digits or level of uncertainty in the measure*; and

A x 10^x

where A represents some number, and x is an exponent (or power) of 10.

b. *The* ***10**^{X}* *part tells what to do with the decimal point. When the exponent ^{X} is a positive number, the decimal point in* ***A*** *goes to the right ^{X} places, if the exponent is negative, the decimal point moves to the left ^{X} places. If the exponent is zero, leave the decimal point alone*.

Scientific Notation	Exponent means move decimal point	Normal Notation
1.23×10^{2}	Two places to the right	123
1.23×10^{-1}	One place to the left	0.123
1.23×10^{0}	Leave alone	1.23

29. Advantages of Scientific Notation. The main reasons for use of scientific notation are:

a. Very large or very small numbers can be written more compactly. Arithmetical operations with them are greatly simplified;

b. An estimate of the result of an involved computation is obtained quickly; and

c. It is used to indicate the precision of measurements. The measurements in the scientific format are recorded with the correct number of significant figures.

30. Conversions To and From Scientific Notation. Often, when converting a number to and from scientific notation, moves the decimal point beyond the beginning or end of the number. This handled as shown below:

Example. Convert 4.6×10^{4} to standard notation. This means moving the decimal point 4 places to the right, ie, 4.6_ _ _. These ‘empty’ places should be filled with zeros ie, **46 000**.

Example. Convert 4.6×10^{-4} to standard notation. This means moving the decimal point 4 places to the left, ie, ._ _ _ 4.6. These ‘empty’ places should be filled with zeros ie, **.00046**.

OPERATIONS WITH SCIENTIFIC NOTATION

31. Addition and Subtraction.

Before adding or subtracting two numbers expressed in scientific notation, their powers of 10 must be equal.

Example: Add 1.63×10^4 and 2.8×10^3. To convert 2.8×10^3 to $A \times 10^4$, move the decimal point one place to the left, $2.8 \times 10^3 = 0.28 \times 10^4$, and add the numbers:

1.63

+ 0.28

1.91 the answer is **1.91×10^4**

Example: Subtract 5.2×10^{-5} from 9.347×10^{-3}. To convert 5.2×10^{-5} to $A \times 10^{-3}$, move the decimal point two places to the left, $5.2 \times 10^{-5} = 0.052 \times 10^{-3}$, now subtract the numbers:

9.347

- 0.052

9.295 the answer is **9.295×10^{-3}**

32. Multiplication and Division.

To multiply two powers of 10, add their exponents.

Example: Multiply 10^2 by 10^3. = 10^{2+3} = **10^5**

To divide two powers of 10, subtract the exponent of the divisor form the exponest of the dividend.

Example: Divide 10^6 by 10^4.

10^{6-4} = **10^2**

To multiply two numbers expressed in scientific notation, multiply separately their decimal parts and their powers of 10.

Example. Multiply 1.245×10^3 by 2.70×10^2.

$(1.245 \times 2.70) \times 10^{3+2} = 3.3615 \times 10^5$ or $\mathbf{3.36 \times 10^5}$. The answer must rounded off to the nearest significant digit to match the least precise number which is 2.70 which only has two significant figures to the right of the decimal place.

To divide two numbers expressed in scientific notation, divide separately their decimal parts and their powers of 10

Example. Divide 6.75×10^5 by 1.5×10^3.

$(6.75 \div 1.5) \times (10^{5-3}) = \mathbf{4.5 \times 10^2}$

Example. Divide 8.5×10 by 1.25×10^{-2}.

$(8.5 \div 1.25) \times (10^{1-(-2)}) = 6.8 \times 10^{1+2} = \mathbf{6.8 \times 10^3}$
(note that the sign change in the divisor which is added to the dividend’s exponent, the same as in subtraction and division of negative numbers)

Practical Exercise

10. Convert the following measured values from scientific notation to normal notation. For each one, indicate the number of significant figures.

a. 5.60×10^{1} kg

b. 2.5×10^{-4} m

c. 5.600×10^{6} km

d. 0.02×10^{2} m

11. Convert the following numbers to scientific notation.

a. 226
b. 226.0
c. 0.000 000 000 50
d. 0.3
e. 0.30

f. 900 000 574 with an uncertainty of $\pm$ 0.5 million
g. 900 000 574 with an uncertainty of $\pm$ 50

12. Multiply 2.4×10^{6} by 6.25×10^{-4}

13. Calculate $\frac{1.56 \times 10^{2} \times 2.0 \times 10^{8}}{5.2 \times 10^{5}}$

14. The toxicity of a nerve agent is measured to be 17 parts per million.

a. Express this in scientific notation.
b. Write the answer in ordinary decimal notation.

DIMENSIONAL ANALYSIS

33. Dimensions are an important aspect of any scientific or technological investigation, problem or experiment. They are part of the language used to express relationships, events and results. *Dimensions refer to the physical characteristic or quantity of an entity*. Time is a dimension, length, mass, velocity and temperature are other examples of dimensions. In solving scientific or technology problems, it is essential that the derived formulae, correct formulae and the correctness of the result are expressed in the correct dimensional units. Dimensions can be treated as algebraic quantities. This means that before you can add, subtract, multiply, divide or manipulate an equation or numbers, they must be in the same dimensions. Furthermore, the terms on both sides of an equation must be expressed in the same dimensions. For example, you cannot multiply time and length to determine the area of an object. You cannot determine the area of a field by multiplying the length by the width if one dimension is in yards and the other is in metres. You must convert one of the dimensions so that both are either in yds or m. *Dimensional analysis is simply a conceptual tool or method of using known units in a problem to arrive at a solution with the correct dimensions.* Sometimes it is called *unit analysis*.

> **Dimensional analysis** is a method for determining the units of a variable in an equation

34. Steps. The steps in dimensional analysis are:

a. *Write down the given measurement and its units*. You can always recognize a conversion factor because it has two different units one over the other, ie 60 s = 1 min.

b. *Multiply the measurements by the appropriate conversion factor so that the original units of the measurement cancel out.* Make sure that when using conversion factors, the units you want to 'disappear' are always arranged so that they cancel.

c. *Perform the calculation.*

35. Making Dimensional Analysis Work. Dimensional analysis is frequently encountered in converting units ie, miles to kilometers and in mathematical calculations in physics and chemistry.

Example. How many seconds (s) are there in 2.0 years?

The first step is to make sure the dimensions are the same ie, secs (s).

$$? \text{ s} = 2.0 \text{ yr} \times \frac{365 \text{ days}}{1 \text{ yr}} \times \frac{24 \text{ hrs}}{1 \text{ day}} \times \frac{60 \text{ min}}{1 \text{ hr}} \times \frac{60 \text{ s}}{1 \text{ min}}$$

$? \text{ s} = 2.0 \times 365 \times 24 \times 60 \times 60 \text{ s} = 63\ 072\ 000 \text{ s}$ or

$= \mathbf{6.3 \times 10^7\ s}$ (to 2 significant figures since that is the least precise measure (2.0) in the numbers)

Example. How many centimeters are there in 6.00 in, given that 1 in = 2.54 cm?

The first step is to make sure the dimensions are the same ie, cm.

? cm = 6 .00 ~~in~~ x 2.54 cm = 15.2 cm (to 3 significant figures)

1 ~~in~~

Example. A convoy traveling at 45.0 kph has to make a trip of 100.0 km. How many min will the trip take? We know that speed (s) = distance(d) ÷ time (t) (s=d / t). Manipulating this formula by multiplying both sides of the equation by time, we get

$$speed = \frac{distance}{time}$$

$$speed * time = \left(\frac{distance}{time}\right) * time$$

$$speed * time = \left(\frac{distance}{\cancel{time}}\right) * \cancel{time}$$

dividing both sides of the equation by speed, we get

$$\frac{(speed*time)}{speed} = \frac{distance}{speed}$$

$$\frac{(\cancel{speed} * time)}{\cancel{speed}} = \frac{distance}{speed}$$

$$time = \frac{distance}{speed}$$

$$t = 100\text{ km} \div 45 \frac{k}{hr} = \frac{100\cancel{k} \times hr}{45\cancel{k}} = 2.22\cancel{hr} \times \frac{60\text{ min}}{\cancel{hr}} = \mathbf{133.2\ min}$$

Practical Exercise

15. Mach number is a term used by pilots and missileers to describe the velocity of an aircraft or missile. It is a dimensionless number giving the ratio of the velocity of the object compared to the speed of sound in air, at a temperature of 15 degrees Celsius and at sea level. The speed of sound is 761.2 mph. A non-tech trained officer states that a anti-tank missile is faster than any tank round fired from a Leopard II C1 tank. The missile's velocity is mach 3, the tank's APFSDS round's velocity is 2000 m/s, its HEAT round is 1425 m/s and its MPHE round is 960 m/s. Which is faster, the missile or the tank rounds? One mile = 1.609 km.

16. You are the leader of a small reconnaissance patrol that is being launched by kayak off shore. You must be ashore with at least one hour of darkness left so that you have time to hide your boats. The submarine captain tells you because of the shallow depths, he cannot venture closer than 12 nautical miles (NM) from shore. Your boat crews can average about 4 miles per hr (4mph) in the ocean swell which is expected to be present. It is now 0300 hrs, dawn is at 0600 hrs. Can you make it? A nautical mile is 1,852 metres(m).

17. You are completing a helicopter recce of an operational area in a British Lynx helicopter armed with a full load of hellfire missiles. It has a 150 kt transit speed. The pilot tells you that a message has come in to return to your forward operating base immediately because it is under attack and needs the helicopter's fire support within the next 20 min. You are 100 km away. Can you make it in time? A knot (kt) is equal to one nautical mile per hour.

PROBLEM SOLVING STRATEGY

36. There are many problem solving approaches and methodologies that AMS students will be introduced to. This section is intended to provide an approach to help students solve mathematical problems encountered in the science and technology courses.

37. The approach below is intended to help understand the problem, organize work and provide a direction in solving a scientific problem. The steps in the approach are:

a. Read the Problem. Read the problem carefully. Past experience with AMS student results on exams, tests, assignments and exercises indicates that this is the major problem. Students tend to gloss over the question without capturing all of the necessary information (and hints). Be sure you understand the nature of the problem and what is being asked and what information is being provided before attempting to solve the problem.

b. Visualize. Visualization is simply clarifying in your mind what is happening and identify where gaps in the information you are given exists (these gaps are usually the things you need to find answers to). It consists of two sub-steps.

(1) Draw a Diagram. Draw a suitable diagram/picture of the problem with labels, axes and coordinates to allow you to picture the problem, define its scope and limits and the information that has been provided; and

(2) Visualization. Attempt to visualize what is happening as defined by the problem and information it gives you.

c. Identify the Principles. Identify the basic scientific principles that are involved and affecting the actions that could take place. List the knowns and unknowns.

d. Choose Equations. Select the relationships or equations that can be used to find the unknowns and algebraically solve the equation with the knowns for these unknowns.

e. Solve the Equations. Substitute the given values (knowns) with the appropriate units into the equation. Manipulate the formulae/equations to obtain the results.

f. Evaluate and Check. Do the algebra or math and obtain a numerical value for the unknowns. Do the units match? Is the answer reasonable? Is the algebraic sign + or – correct or meaningful?

PRACTICAL EXERCISE ANSWERS

PRECISION AND ACCURACY.

1. a. Rumbles was most accurate because 2792 is closest to the true count of 2800 words in the essay. The next most accurate would be Bumbles with 2810.

   b. Because each person made only one count, precision does not apply. More than one count is necessary.

   c. The average word count is 2736 + 2792 + 2810 + 2734 ÷ 4 = 2768. This is a difference of 32 words fewer than the actual number but because 32 is quite small compared to 2800 (just over 1%) the group's count can be considered accurate.

   d. The average word count 2722 + 2724 + 2719 + 2723 ÷ 4 = 2722 or 78 words fewer than the first group. Therefore they are less accurate than the first group. In terms of precision, the count spread was from a high of 2724 to a low of 2719 or a difference of only 5 words. The first group's spread was from a high of 2810 to a low of 2734 words or a difference of 76 words. Therefore, the second group is more precise.

2. Jones' average is 256 + 263 + 262 + 266 ÷ 4 = 261.75 rounded off to 262
   Smith's average is 250 + 242 + 270 + 278 ÷ 4 = 260

   a. Smith is more accurate because his average of 260 is closest to the actual number of 260.

   b. Jones is more precise because his spread ranges from 256 to 266 or a difference of 10, while Smith's spread is 242 to 278 or a difference of 36 rds.

UNCERTAINTY AND SIGNIFICANT NUMBERS.

3. The last digit written is assumed to be uncertain, and the uncertainty is determined by putting a 1 in the place of the uncertain digit and then dividing by 2.

4. a. 14.50 $\pm$ 0.005 mm

   b. 14.5 $\pm$ 0.05 mm

   c. 0.000 000 03 $\pm$ 0.000 000 005 in

   d. 125 $\pm$ 0.5 mm

5. a. 5.56 ± 0.005 mm

b. 155 ± 0.5 mm

c. .50 ± .005 in

SIGNIFICANT NUMBERS

6. a. 3 significant figures

b. 4 significant figures

c. 3 significant figures

d. 2 significant figures

7. a. 4 significant figures

b. 1 significant figure

c. 4 significant figures

d. 2 significant figures

e. 5 significant figures

OPERATIONS WITH SIGNIFICANT NUMBERS.

8. a. Since 4.575 has uncertainty in the third decimal place, 0.0926 must be rounded off to the nearest 0.001 and becomes 0.093. Subtracting: 4.575 – 0.093 = 4.482.

9. 38.44 + 38.44 + 19.5 + 19.5 = 115.88 but since the least precise measurement (19.5 m) is significant to only one decimal place, the answer must be rounded to the nearest decimal place = 115.9 m.

OPERATIONS WITH SCIENTIFIC NOTATION.

10. a. 56.0 kg , 3 significant figures

b. 0.000 25 , 2 significant figures

c. 5 600 000 km , 4 significant figures but you can' tell from normal notation, only the scientific notation

d. 2 m , 1 significant figures

11. a. 2.26×10^2

b. 2.260×10^2

c. 5.0×10^{-10}

d. 3×10^{-1}

e. 3.0×10^{-1}

f. 9.00×10^8 when rounding to $\pm$ 0.5 million, the answer becomes 0

g. $9.000\,006 \times 10^8$

12. Multiply, separately, the two parts:

$2.4 \times 6.25 = 15.0 = 1.50 \times 10^1$

$10^6 \times 10^{-4} = 10^2$

The product is $1.50 \times 10^1 \times 10^2 = 1.50 \times 10^{1+2} =$ $\mathbf{1.5 \times 10^3}$

13. $\frac{1.56 \times 2.0}{5.2}$ x 102+8-5 = 0.60×10^5 or 6.0×10^4

14. a. 1 million = 10^6, therefore $\frac{17}{10^6} = 1.7 \times 10^1 \times 10^{-6} =$ $\mathbf{1.7 \times 10^{-5}}$

b. $1.7 \times 10^{-5} = 0.000\,017$

DIMENSIONAL ANALYSIS AND CONVERSIONS.

15. The first step is to convert the mach no into mph. 3 x 761.2 mph = 2283.6 mph, next this figure needs to be converted into m/sec. Knowing that 1 mi = 1.609 km, that 1 km = 1000 m, 1 hr = 60 min and 1 min = 60 s, rearrange the conversion factors so as to replace mi by m and hr by min.

$$\frac{2283.6\ \text{mi}}{1\ \text{hr}} \times \frac{1.609\ \text{km}}{1\ \text{mi}} \times \frac{1000\ \text{m}}{1\ \text{km}} \times \frac{1\ \text{hr}}{60\ \text{min}} \times \frac{1\ \text{min}}{60\ \text{s}} = \frac{2283.6 \times 1.609 \times 1000}{60 \times 60} = 1020.6\ \text{m/s}$$

Therefore the missile is not faster than two of the tanks rounds and it is just barely faster than the tank's slowest round.

16. We know from the previous question that 1 mi = 1.609 km. The first step is to convert nautical miles into m and mph into m/hr

$$12\ \text{NM} \times \frac{1852\ \text{m}}{1\text{NM}} = 22{,}224\ \text{m}$$

$$\frac{4\ \text{mi}}{1\ \text{hr}} \times \frac{1.609\ \text{km}}{\text{mi}} = 6.436\ \text{kph or } 6.436 \times 1000 = 6436\ \text{m/hr}$$

To find how long it will take, divide the distance by the speed

$$\frac{22,224\ \text{m}}{6436\ \text{m/hr}} = \frac{22,224\ \text{m} \cdot \text{hr}}{6436\ \text{m}} = 3.453\ \text{hrs} = 3.5\ \text{hrs}$$

Therefore you can't make it in time and must either wait for the next night or have the Capt place you closer to shore!

17. The first step is to convert the knots into kph:

$$\frac{150\ \text{NM}}{1\ \text{hr}} \times \frac{1852\ \text{m}}{1\ \text{NM}} \times \frac{1\ \text{km}}{1000\ \text{m}} = \frac{150 \times 1852}{1000}\ \frac{\text{km}}{\text{hr}} = 277.8\ \text{kph}$$

to determine the time to travel the 100 km

$$\frac{1\ \text{hr}}{277.8\ \text{km}} \times \frac{60\ \text{min}}{1\ \text{hr}} \times 100\ \text{km} = \frac{6000}{277.8}\ \text{min} = 21.598\ \text{min rounded to one}$$

Significant digit = **21.6 min**. Therefore you can't make it in time.

CHAPTER 3 - SELF-TEST

1. Mathematics. This self-test is based on simple mathematics. Students should:

a. Complete the self-test questions for Mathematics on the following pages. Complete each self-test question in which you fell confident and score it yourself, one (1) mark per correct answer. The answers to each test can be found at the end of this chapter;

b. If you have difficulty with the questions (if your score less than 70% in the section) it is strongly recommended you learn that topic by working through the appropriate chapter in the text books, review and study the contents, and practice by completing the self-tests and exercises at the end of the appropriate Chapter/section in the text;

c. Once you satisfactorily understand the material, return and complete that portion of this self-test; and

d. If you still have difficulty completing the test, call the advisor identified in your joining instructions.

WHOLE NUMBERS

1. 1 RCR had three LAV III vehicles that weighed 18,234, 17,645 and 19,821 kg. What is the total weight of all three vehicles?

2. Pte Smith walks at 7 km/hr and Pte Jones walks at 9 km/hr. If they both walk continuously for 9 hours along the same route, what is the distance between them? If they walk in exactly opposite directions, what is the distance between them?

3. What is the sixth power of three, ie, 3^6?

4. There are 6400 mils in a circle. If I divide the circle in eight equal arcs, how many mils are in each arc?

FRACTIONS

1. What is the lowest common denominator of 1/3, 1/8, 1/5 and 1/16?

2. Divide 3/4 by 1/6 and reduce to the lowest terms possible.

3. Joe travels 636 km in 8 hours on Monday. It took him the same time to travel 1/2 the distance on Tuesday. It took him the same time to travel 1/3 the distance on Wednesday. How far would he have to travel on Thursday to go 1/2 of the total of Tuesday's and Wednesday's combined travel?

4. Pte Smith walks at 7 1/2 km/hr and Pte Jones walks at 9 1/3 km/hr. If they both walk continuously for 9 hours along the same route, what is the distance between them? If they walk in exactly opposite directions, what is the distance between them?

DECIMALS

1. Write 17/32 as a decimal.

2. Add 6.17 and 4.26 and 3.56 then multiply the sum by 2.3

3. Divide 3.18 by 0.53

4. Pte Smith walked 7.482 km, Pte Jones walked 9.613 km and Pte Lewis walked 6.242 km, what is the average distance that the three Ptes walked?

PERCENTS

1. What is 24% of a 16,400 kg elephant?

2. Vehicle X weighs 12,450 kg. Its weight has to be reduced by 12%, what is its new maximum allowable weight?

3. A 7.62 mm bullet weighs 9.53 grams. The bullet consists of a lead antimony core coated with a gilding metal jacket. The gilding metal jacket is 14% of the total weight. What is the weight of the lead antimony core?

4. If Pte Smith walks 10 km in 1 2/3 hrs, how long will it take him to walk 36 km?

SIGNED NUMBERS

1. Divide –144 by –12
2. Find the sum of –62, 14, 12 and –36
3. Find the product of –14 and 17
4. Evaluate –64 – 17

ALGEBRAIC EXPRESSIONS

Express the following in a mathematical expression:

1. Multiply two hundred and sixteen by minus twelve.

Add two hundred and twelve to the product of eight and six.

Solve for x:

3. $6x + 14 = 32$

4. $4x + 13 = 6x + 1$

POLYNOMIALS

Factor the following completely:

1. $6x^2 + 8x = 0$

2. $x^2 - 1 = 0$

3. $x^2 - x - 20 = 0$

4. $6x^2 - 9x - 42 = 0$

LINEAR EQUATIONS

Solve for x and y:

1. $3x - y = 6$
   $2x + 3y = 26$

2. $x + 2y = -5$
   $4x - 2y = 20$

3. $2x + 4y = -32$
   $x - 3y = 14$

QUADRATIC EQUATIONS

What are the roots of the following quadratic equations:

1. $x^2 + 6x + 8 = 0$

2. $x^2 + 2x - 15 = 0$

3. $6x^2 - 8x - 8 = 0$

4. $x^2 - 2x + 1 = 0$

SEQUENCES AND SERIES

Fill in the missing number from the following sequences:

1. 3, 6, 9, ___, 15

2. 1, 2, 3, 5, ___, 13

3. 3, 7, 6, ___, 9, 13, 12

Sum the following sequence:

4. 6, 9, 15, 24, 39, 63

GEOMETRY

1. What is the supplementary angle of 64°?

2. What is the perimeter of a rectangle that has sides of 1.3 meters and 4.4 meters?

3. The side of the right triangle that is opposite the right angle is called what?

4. The LAV III tires have a 0.65 meter radius. What is the circumference of the tire? (to 4 decimal places)

MEASUREMENT OF GEOMETRIC FIGURES

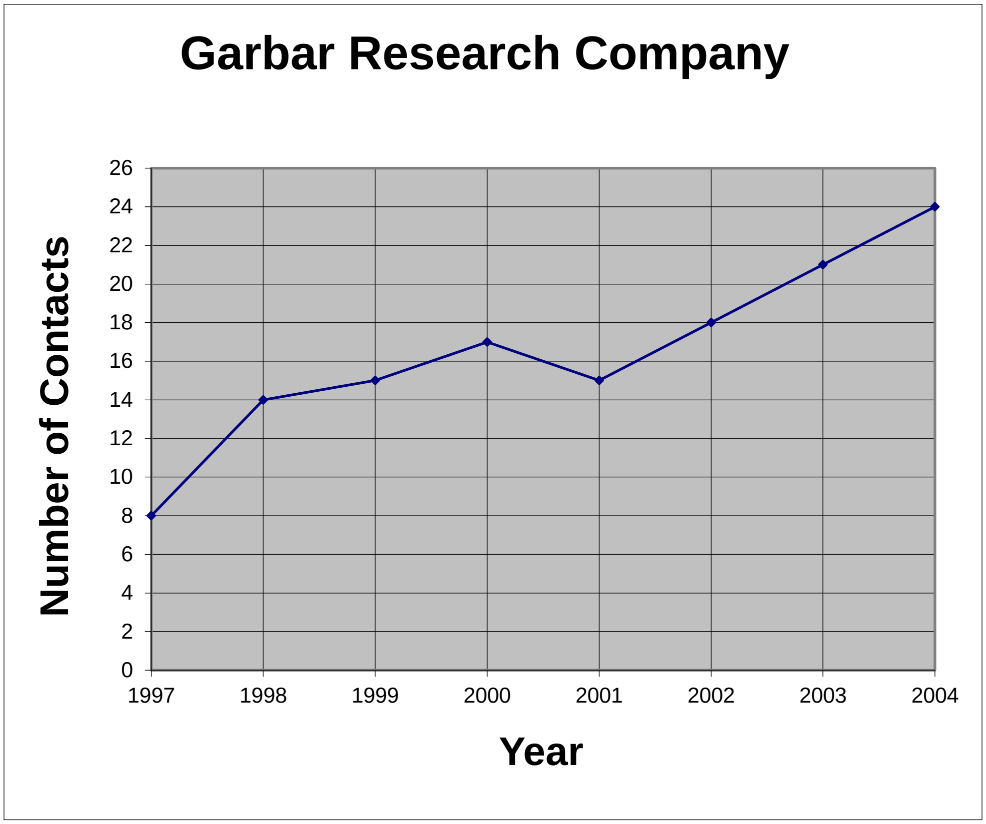
1. What is the area of a square with a perimeter of 48 meters?

2. What is the diameter of a circle with an area of 113.04 cm^2?

3. What is the volume of a pyramid with a 6 cm x 6 cm square base and a 12 cm height?

4. What is the volume of a sphere is 904.32 mm^3. What is its diameter?

GRAPHS



Using the above graph:

1. Which year(s) did Garbar reach 15 contacts?

2. How many years did Garbar have more than 15 contacts?

3. What is the total number of contracts Garbar has between and including 1997 and 2004?

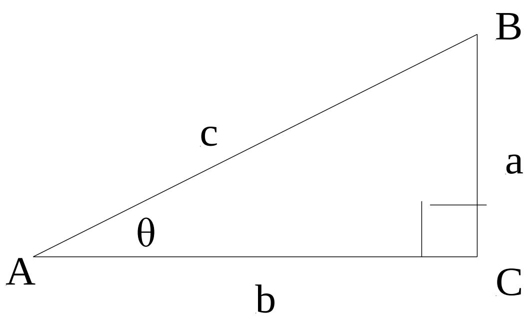
PROBABILITY

1. How many different signals can a forward operating base make by hoisting five (5) different flags in a line to a signal mast?

2. How many different selections can there be made of one or more soldiers for a patrol from a detachment of 16 men?

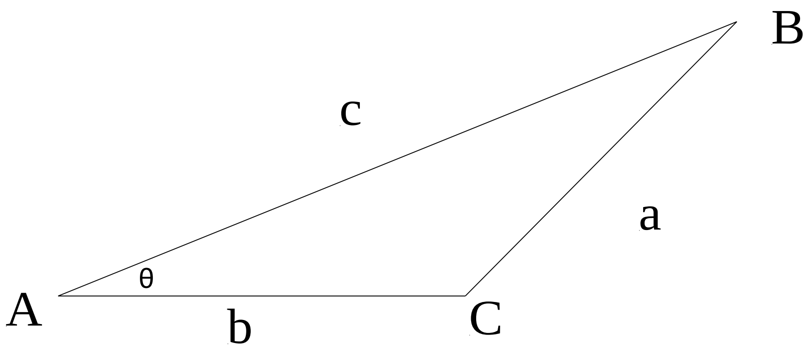
3. There has been a major error made in production of grenades, dummy and live grenades were painted and marked identically and shipped into theatre. Two of the mixed grenade boxes have arrived at your FOB. Each of two boxes contains ten grenades which are identical except that in each case, four are dummies and six are live. What is the probability that if two grenades are selected at random from each box, all four turn out to be dummies?

TRIGONOMETRY



1. Given a = 12 cm and b = 9 cm, what is the length of c?

2. Given a = 6 m and c = 17 m, what is the angle A or θ?



3. Given a = 12 mm, b = 15 mm and B = 22.5°, find c.

4. Given b = 45 cm, c = 76.1 cm and A = 17.8°, find a.

CHAPTER 3 - SELF-TEST – SOLUTIONS

WHOLE NUMBERS

1. 55,700 kg
2. 18 km and 144 km
3. 729
4. 800 mils

FRACTIONS

1. 240
2. 4 1/2
3. 265 km
4. 16 1/2 km and 151 1/2 km

DECIMALS

1. 0.53125
2. 32.177
3. 6
4. 7.779

PERCENTS

1. 3936 kg
2. 10,956 kg
3. 8.1958 grams
4. 6 hours

SIGNED NUMBERS

1. 12
2. –72
3. –238
4. –81

ALGEBRAIC EXPRESSIONS

1. $216 \times (-12) = -2592$
2. $212 + 8 \times 6 = 260$
3. x = 3

4. $x = 6$

POLYNOMIALS

1. $2x(3x + 4)$
2. $(x + 1)(x - 1)$
3. $(x + 4)(x - 5)$
4. $(2x - 7)(3x + 6)$

LINEAR EQUATIONS

1. $x = 4, y = 6$
2. $x = 3, y = -4$
3. $x = -4, y = -6$

QUADRATIC EQUATIONS

1. $x = -2$ or -4
2. $x = 3$ or -5
3. $x = 2$ or $-2/3$
4. $(x-1)^2$ roots 1,1

SEQUENCES AND SERIES

1. 12
2. 8
3. 10
4. 156

GEOMETRY

1. 116°

2. 11.4 meters

3. hypotenuse

4. 4.0841 meters

MEASUREMENT OF GEOMETRIC FIGURES

1. 144 m^2

2. 12 cm

3. 144 cm^2

4. 12 mm

GRAPHS

1. 1999 and 2001

2. 4

3. 132

PROBABILITY

1. 120

2. $M = 2^{16} - 1 = 65{,}536-1 = 65{,}535$

3. 4/255

TRIGONOMETRY

1. 15 cm

2. 20.7°

3. 25.37 mm

4. 36 cm

CHAPTER 4 - SELF-TEST – PART ONE

1. <u>Mathematics</u>. This self-test is based on simple mathematics. Students should:

   a. Complete the self-test questions on the following pages. Complete each self-test question in which you fell confident and score it yourself, one (1) mark per correct answer. The answers to each test can be found at the end of this document;

   b. This portion of the self test should be conducted without a calculator until Section 3;

   c. If you have difficulty with the questions it is strongly recommended you learn that topic by returning to the appropriate section in the excerpted document, review and study the contents.

   d. Once you satisfactorily understand the material, return and complete that portion of this self-test; and

   e. If you still have difficulty completing the test, call the advisor identified in your joining instructions.

SECTION 1: WHOLE NUMBERS

1. Karen and Steve hiked 52 miles in 5 days. The first day they hiked 12 miles, the second day 7 miles, the third day 13 miles and the fourth day 9 miles. How many miles did they hike on the last day?

2. Two machinists operating the same lathe work 8 hours each on a day- and night-shift respectively. The day-shift guy turns out 35 pieces per hour and the night-shift guy turns out 50 pieces per hour. What will be the difference in their output at the end of 30 days?

3. Find the cube of 7.

SECTION 2: FRACTIONS

1. Solve the following fraction problems. Reduce to lowest terms where possible:

a. $1\frac{1}{2}+3\frac{5}{8}=$

b. $\frac{52}{56}-\frac{3}{7}=$

c. $\frac{1}{6}\times\frac{13}{17}=$

d. $\frac{3}{4}\div\frac{3}{29}=$

2. The distance between New York and California is 3000 miles. Two trains leave the two cities at the same time. One train travels at the rate of $54\frac{1}{3}$ miles an hour, the other at $57\frac{3}{5}$ miles an hour. How far apart will the two trains be at the end of 5 hours?

SECTION 3: DECIMALS

1. A 16-story apartment building is 163.86 feet high. How high is the ceiling of the fifth floor from the ground?

2. Write 0.1875 as a fraction.

SECTION 4: PERCENTS

1. How much is $62\frac{3}{4}$ % of $75?

2. What number increased by 20% of itself equals 240?

SECTION 5: SIGNED NUMBERS

1. Solve the following signed number problems:

   a. Subtract -77 and -92 from 56.

   b. Evaluate (-12 x 16) + -4.

   c. |137 – 45 x 11| =

SELF-TEST – PART ONE SOLUTIONS

SECTION 1: WHOLE NUMBERS

1. 11 miles.

2. The night shift guy will turn out 3600 more pieces in 30 days than the day shift guy.

3. 343

SECTION 2: FRACTIONS

1. HINT: Don't forget to reduce to lowest terms!

   a. $5\frac{1}{8}$

   b. $\frac{1}{2}$

   c. $\frac{13}{102}$

   d. $7\frac{1}{4}$

2. $240\frac{1}{3}$ *miles*

SECTION 3: DECIMALS

1. 51.206 feet

2. $\frac{3}{16}$

CHAPTER 5 - SELF-TEST – PART TWO

1. Mathematics. This self-test is based on mathematics that you will see on the course. Questions are based on the knowledge gained by completing the required study package, above Students should:

a. Complete the self-test questions on the following pages. Complete each self-test question in which you fell confident and score it yourself, one (1) mark per correct answer. The answers to each test can be found at the end of this document;

b. Calculators should be used sparingly however one will be required for trigonometry, exponentials and logarithms;

c. If you have difficulty with the questions it is strongly recommended you learn that topic by returning to the appropriate section in the provided text and study the contents.

d. Once you satisfactorily understand the material, return and complete that portion of this self-test; and

e. If you still have difficulty completing the test, call the advisor identified in your joining instructions.

2. Please keep in mind that the material in this part of the self-test will be reviewed more in-depth with an instructor during the course of the summer term. This self-study package is designed to give you advanced warning as to what can be expected during the math program. It also gives you the opportunity to give yourself a "leg up" or advantage prior to starting the AMS program. If you are having extreme difficulties on particular sections, **do not get discouraged**, take note of the issue and present it to the instructor upon commencement of the course.

SECTION 1: POWERS, ROOTS & RADICALS

1. Simplify the radicals:

a. $\sqrt[3]{-27x^{10}y^{8}}$

b. $\sqrt{75x^{2}y^{3}}$

2. Rewrite the following expression as an integer (number):

a. $8^{\frac{7}{3}}$

b. $9^{\frac{4}{3}}$

3. Simplify the radical expressions, **rationalizing** if necessary:

a. $\sqrt{12x} - \sqrt{48x}$

b. $\sqrt[3]{54x^5} + \sqrt[3]{16x^5}$

c. $\left(\sqrt[3]{9x^2y}\right)\left(\sqrt[3]{6x^2y^2}\right)$

d. $\sqrt{35x^2y^7} \div \sqrt{7xy^8}$

SECTION 2: BASIC ALGEBRAIC FUNCTIONS

1. Classify the following polynomial based on its degree and number of terms:

a. $-7x^4 - 9$

b. $7 + 9x - 4x^5 + 3x^3$

2. Calculate the products of the following polynomials and simplify:

a. $5x^3(2xy + 7x - 3y) - 2xy(y^3 + 2x^3 - 8x)$

b. $(2x^2 - 9xy^3)(xy + 7y^2)$

3. Divide the following polynomials using the long division method:

a. $(x^4 - 25x^2 + 62x - 36) \div (x^2 - 3x - 18)$

b. $(4x^3 + 32) \div (x + 2)$

SECTION 3: LINEAR EQUATIONS, INEQUALITIES & GRAPHING

1. Solve the following linear equations.

   a. $-3(x+7)=-2(x-1)+5$

   b. $2|x-3|+5=13$

2. Solve the following systems of equations. Find the values of both x and y.

   a. $3x+y=1$
   $4x+3y=-2$

   b. $x-3y=-4$
   $3x-9y=-11$

3. Solve the following linear inequalities. Show the solution on a number line.

   a. $14-3(y+5)>8$

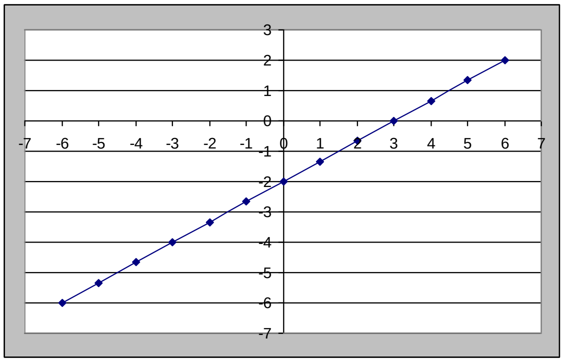
   b. $|2x+3|-8\geq 0$

4. What are the coordinates of the x- and y-intercept for the following linear equation:

$$4x-3y=8$$

5. Which of the equations below has the following graph?

   Hint: Point-to-point plotting is a waste of time if you can put the equation into standard form.



a. $2x+3y=6$

b. $-2x+3y=6$

c. $2x-3y=6$

d. $2x+3y=-6$

6. Write the equation of the line that passes through the points: (4,3) and (-9,2).

7. Determine the equation of the line with the slope m = $-\frac{2}{3}$ that passes through the point (-1,2) and write it in standard form.

SECTION 4: QUADRATIC & HIGHER ORDER POLYNOMIALS

1. Factor the following polynomials:

a. $12x^2y - 6x^2y^2 + 9x^5y^3$

b. $6x^2 - 150$

c. $2x^2 + 11x - 6$

d. $8x^3 - 64$

e. $2x^3 - 8x^2 + 6x$

f. $10x^2 + 15x - 12xy - 18y$

2. Solve the following higher order equations. Find all values for x. If the solution for x is irrational put it in the simplest radical form.

a. $4x^4 + 4x^3 - 9x^2 - x + 2 = 0$

b. $2x^3 - 11x^2 + 4x + 5 = 0$

SECTION 5: SOLVING RATIONAL EXPRESSIONS

1. Simplify the expressions:

a. $\frac{x^2 - 8x + 15}{x^2 - 25}$

b. $\frac{x^2 + 3x - 15}{x^2 + 8x + 15}$

2. Rewrite each as a single expression and simplify:

a. $\frac{3x}{x^2 - 9x + 20} - \frac{2}{x - 5}$

b. $\frac{x^2 y^3}{x^2 + 13x + 36} \times \frac{x^2 - 81}{x^4 y^2}$

c. $\frac{x^2 - 6x - 27}{x^2 - 11x + 18} \div \frac{x^3 + 2x^2 - 3x}{x^2 - 2x}$

SECTION 6: QUADRATIC INEQUALITIES & GRAPHING

1. Solve the following inequalities and graph the solutions on a number line.

a. $x^2 + 2x \leq 24$

b. $-12x^2 - 5x + 3 > 0$

c. $\dfrac{3x+1}{x-4} \le 2$

2. Sketch the following functions on a coordinate plane without using point-by-point plotting.

a. $y = x^2 - 6x + 5$

b. $16x^2 - 64x + y^2 + 2y + 49 = 0$

c. $x^2 + y^2 + 8x - 2y = -8$

SECTION 7: LOGARITHMS & EXPONENTIALS

1. Express the given logarithmic equation exponentially. If required, solve for the unknown variable.

a. $5 = log_2 32$

b. $log_b 81 = 4$

2. Express the given exponential equation in logarithmic form.

a. $4^{-3} = \dfrac{1}{64}$

b. $\sqrt{9} = x$

3. Evaluate the given logarithmic functions. Remember it will be easier to solve by converting the expressions into exponential form first. Do not use a calculator for this part.

a. $log_3 1$

b. $log_{11} 11$

c. $log_7 \sqrt{7}$

4. Evaluate the following common and natural logarithmic functions.

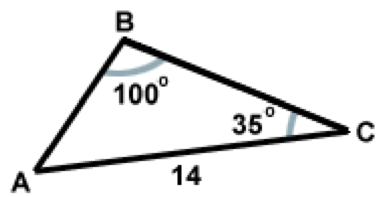
a. $10^{log 75}$

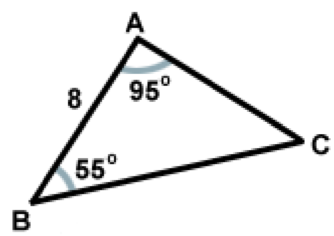
b. $ln\left(\frac{1}{e^5}\right)$

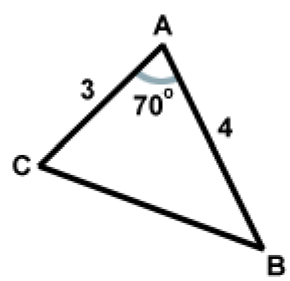
SECTION 8: TRIGONOMETRY

1. On the top of the Sears Building in Chicago is a TV tower. From a point on the ground 300 ft. away the bottom of the Sears Building the base of the TV tower is at an angle of elevation equal to 42.14 degrees and the top of the TV tower is at an angle of elevation equal to 45.81 degrees. How tall is the TV tower?

2. Resolve all sides and angles of the following oblique triangles:

a. 

b. 

c. 

3. Provide the measurement of the following angles in degrees (decimal format), radians and mils.

a. $\frac{7\pi}{6}$

b. 72^{o} 15’ 13”

c. 125 mils

SELF-TEST – PART TWO SOLUTIONS

SECTION 1: POWERS, ROOTS & RADICALS

1. a. $-3x^3y^2\sqrt[3]{xy^2}$

   b. $5xy\sqrt{3y}$

2. a. 128

   b. 18.7 = 19 (NOTE: Decimals are not integers, only $\pm$ whole numbers)

3. a. $-2\sqrt{3x}$

   b. $5x\sqrt[3]{2x^2}$

   c. $3xy\sqrt[3]{2x}$

   d. $\dfrac{\sqrt{5xy}}{y}$

SECTION 2: BASIC ALGEBRAIC FUNCTIONS

1. a. Terms: 2 = Binomial
   Degree: 4 = Quartic
   Quartic Binomial

   b. Terms: 4 = Polynomial
   Degree: 5 = Quintic
   Quintic Polynomial

2. a. $x(6x^3y + 35x^3 – 15x^2y – 2y^4 + 16xy)$

   b. $xy(2x^2 + 14xy – 9xy^3 – 63y^4)$

3. a. $x^2 + 3x + 2 + \frac{122x}{x^2 - 3x - 18}$

b. $4x^2 - 8x + 16$

SECTION 3: LINEAR EQUATIONS, INEQUALITIES & GRAPHING

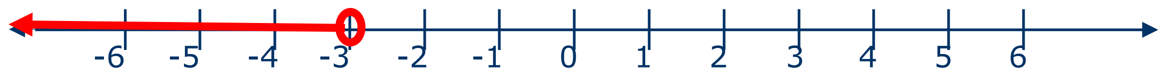
1. a. $x = -28$

b. $x = 7$
$x = -1$

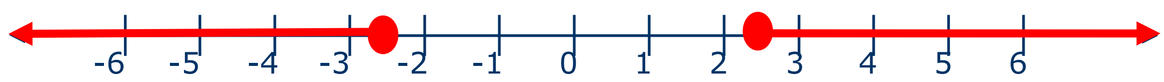
2. a. $x = 1$
$y = -2$

b. No solutions! $0 \neq 1$

3. a. $y < -3$



b. $x \geq \frac{5}{2}$ or $x \leq -\frac{5}{2}$



4. x-intercept = (2 , 0)
y-intercept = $(0 , -\frac{8}{3})$

5. c. $2x - 3y = 6$

6. $y = \frac{1}{13}x + \frac{32}{13}$

7. $y = -\frac{2}{3}x + \frac{4}{3}$

SECTION 4: QUADRATIC & HIGHER ORDER POLYNOMIALS

1. a. $3x^2y(3x^3y^2 - 2y + 4)$

   b. $6(x + 5)(x - 5)$

   c. $(2x - 1)(x + 6)$

   d. $8(x - 2)(x^2 + 2x + 4)$

   e. $2x(x - 1)(x - 3)$

   f. $(5x - 6y)(2x + 3)$

2. a. $(x - 1)(x + 2)(2x - 1)(2x + 1) = 0$

   $x = 1$
   $x = -2$
   $x = \frac{1}{2}$
   $x = -\frac{1}{2}$

   b. $(x - 5)(x - 1)(2x + 1) = 0$

   $x = 5$
   $x = 1$
   $x = -\frac{1}{2}$

SECTION 5: SOLVING RATIONAL EXPRESSIONS

1. a. $\frac{x-3}{x+5}$

   b. $\frac{x^2+3x-15}{(x+3)(x+5)}$

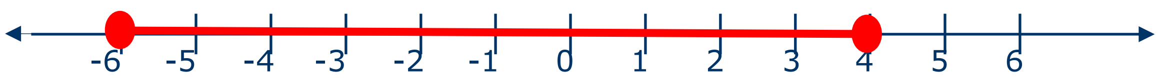
2. a. $\dfrac{x+8}{(x-5)(x-4)}$

b. $\dfrac{y(x-9)}{x^2(x+4)}$

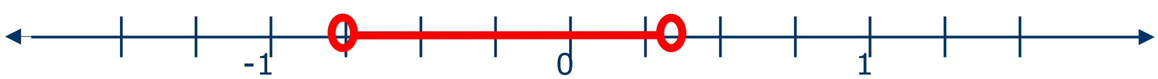
c. $\dfrac{1}{x-1}$

SECTION 6: QUADRATIC INEQUALITIES & GRAPHING

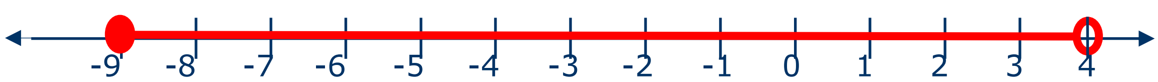
1. a. $-6 \le x \le 4$



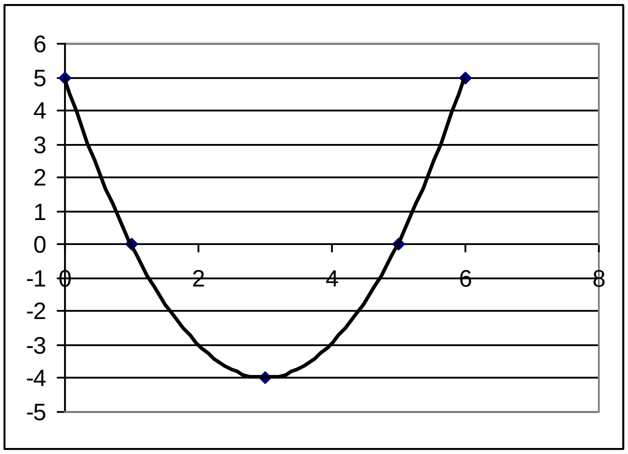
b. $-\dfrac{3}{4} < x < \dfrac{1}{3}$



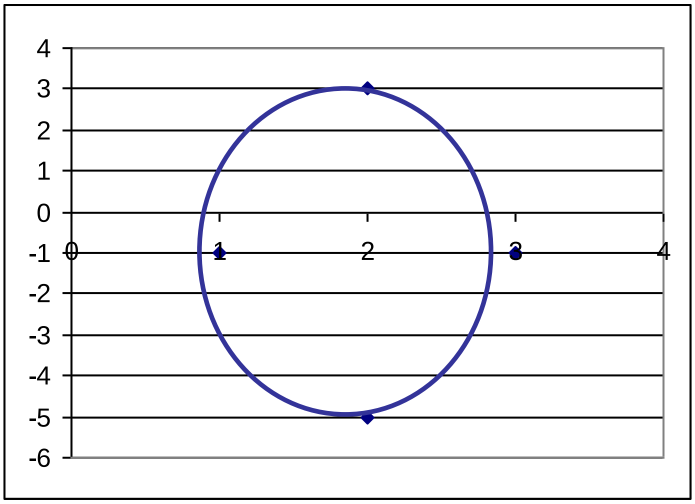
c. $-9 \le x < 4$



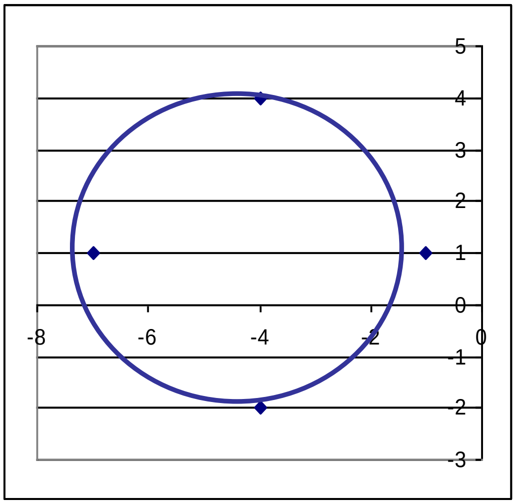
2. a. $y = (x - 3)^2 - 4$ ∴ a **parabola**
minimum point shifts: 3 units right & 4 units down from the origin
x-int: (1 , 0) & (5 , 0)
y-int: (0 , 5)



b. $\frac{(x-2)^2}{1} + \frac{(y+1)^2}{16} = 1$ ∴ an **oval**
center pt: (2, -1)
x-axis radius: 1
y-axis radius: 4



c. $(x+4)^2+(y-1)^2=9 \quad \therefore$ a **circle**
center pt: (-4, 1)
radius: 3



SECTION 7: LOGARITHMS & EXPONENTIALS

1. a. $2^5 = 32$

   b. $b^4 = 81$
   $b = 3$

2. a. $-3 = log_4 \frac{1}{64}$

   b. $\frac{1}{2} = log_9 x$

3. a. 0

   b. 1

   c. $\frac{1}{2}$

4. a. 75

   b. -5

SECTION 8: TRIGONOMETRY

1. 38 ft

2. a. angle A = 45^o
   side a = 12.2
   side c = 8.15

   b. angle C = 30^o
   side a = 15.9
   side b = 13.1

   c. angle B = 43.5^o
   angle C = 66.5^o
   side a = 4.1

3. a. $\frac{7\pi}{6}$, 210^o, 3733 mils

   c. 72^o 15’ 13”, 72.3^o, 1.26 rads, 1285 mils

   c. 125 mils, 7.03^o, 0.123 rads

CHAPTER 6 - STUDY GUIDE – TIMETABLE, LEARNING OUTCOMES, SELF- ASSESSMENT

The following is a timetable for the Pre-Course Study Package. It contains a list of learning outcomes that you should be able to answer. You DO NOT need to go through additional questions if you are comfortable with the material.

<table>
<tr><td colspan="2">REVIEW: Additional online resources – a simple google search on any topic will provide ample learning resources.</td></tr>
<tr><td>Week</td><td>Learning Outcomes</td></tr>
<tr><td rowspan="5">1 – 2</td><td>Chapter 1 – Whole Numbers</td></tr>
<tr><td>On completion of this lesson, you should be able to:
- Explain what are whole numbers
- Add, subtract, multiply and divide whole numbers
- Raise a base number to the given exponent</td></tr>
<tr><td>Chapter 2 – Fractions</td></tr>
<tr><td>On completion of this lesson, you should be able to:
- Explain fractions and prime numbers
- Determine the greatest common divisor
- Add and subtract fractions
- Determine the lowest common denominator
- Multiply and divide fractions
- Simplify fractions
- Explain the order on operations</td></tr>
<tr><td>Chapter 3 – Decimals</td></tr>
</table>

REVIEW: Additional online resources – a simple google search on any topic will provide ample learning resources.	
Week	Learning Outcomes
	On completion of this lesson, you should be able to: - Explain decimals - Convert fractions to decimals - Convert decimals to fractions - Add and subtract decimals - Multiply decimals - Divide decimals
	Chapter 4 – Percents
	On completion of this lesson, you should be able to: - Explain percents - Convert percents to decimal or fractions - Convert decimals to percents - Solve problems involving percents - Solve problems involving ratios and proportions

Week	Learning Outcomes	
1 – 2	Chapter 5 – Signed Numbers	
	On completion of this lesson, you should be able to: - Explain what signed numbers are - Add and subtract signed numbers - Multiply and divide signed numbers - Determine the absolute value of signed numbers	
Algebra & Trigonometry		
Week	Learning Outcomes	
3	Chapter 1 – Fundamental Algebraic Laws and Operations	
	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Explain commutative property - Explain associative property - Explain inverse property - Explain identity property	
	Chapter 2 – Least Common Multiple/Greatest Common Factor/Divisor	
	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Explain Least Common Multiple (LCM) - Explain Greatest Common Factor (GCF)	
4	Chapter 4 – Absolute Values	
	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Explain absolute value properties	
	Chapter 5 – Operations with Fractions	

	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Explain Lowest Common Denominator (LCD) - Multiply and divide fractions - Simplify fractions - Explain the order on operations	
5	Chapter 6 – Base, Exponent, Power	
	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Multiply powers of the same base - Divide powers of the same base - Find the power of a power - Find the power of a product - Find the power of a quotient	

Week	Learning Outcomes	
5	Chapter 7 – Roots and Radicals	
	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Understand how to deal with fractional exponents - Understand how to deal with negative exponents - Introduction to imaginary numbers - Simplify radical expressions	
6	Chapter 8 – Algebraic Addition, Subtraction, Multiplication and Division	
	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Define algebraic expression - Understand how to employ all algebraic operations on expressions with variables. - Understand long division of algebraic expressions	
	Chapter 10 – Solving Linear Equations	
	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Explain systems of linear equations - Solve systems of linear equations by elimination - Solve systems of linear equations by substitution or comparison	
7	Chapter 11 – Properties of Straight Lines	

	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Explain ordered pairs - Graph linear equations - Determine slope and y-intercept from a graph	
	Chapter 12 – Linear Inequalities	
	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Explain the fundamentals of linear inequalities. - Find the complete solution set for an inequality.	
	Appendix 1D – Factoring Expressions and Functions	
	On completion of this lesson, you should be able to: - Explain polynomials and roots - Factor binomials - Factor quadratic polynomials - Factor trinomials	Read through the attached formula sheet.
8	Chapter 15 – Factoring Expressions and Functions	
	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Explain polynomials and roots - Factor binomials - Factor quadratic polynomials - Factor trinomials	

Week	Learning Outcomes	
9	Chapter 16 – Solving Quadratic Equations by Factoring	
	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Explain the standard form of quadratic equations - Explain roots of quadratic equations - Explain completing the square	
	Chapter 17 – Solutions by Quadratic Formula	
	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Explain the quadratic formula	
10	Chapter 18 – Solving Quadratic Inequalities	
	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Explain the fundamentals of quadratic inequalities. - Find the complete solution set for an inequality.	
	Chapter 19 – Graphing Quadratic Equations/Conics and Inequalities	
	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Graph quadratic equations - Graph parabola, circles, ellipses and hyperbola - Graph quadratic inequalities	
11	Chapter 26 – Logarithms and Exponentials	

	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Understand the properties of logarithms - Understand the properties of exponentials	
12	Chapter 27 – Trigonometry	
	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Explain trigonometric functions - Explain the relationship between functions of complementary angles - Use a trigonometric function table - Determine the function of angles - Solve oblique triangles using trigonometric functions	
13	Chapter 28 – Inverse Trigonometric Functions	
	Read through the Basic Attacks and Strategies On completion of this lesson, you should be able to: - Explain inverse trigonometric functions	

CHAPTER 7 - FACTORING POLYNOMIALS – RULES AND EXAMPLES

Factoring is the process of taking what was once a product and breaking it into the original pieces (or factors).

$$\underbrace{x^2 + 2x - 15}_{\text{Product}} = \underbrace{(x - 3)(x + 5)}_{\text{Factors}}$$

Greatest Common Factors (GCF)

Greatest Common Factor – the largest monomial that divides evenly into each term of a polynomial.

The first step in factoring ALL polynomials!

Finding the GCF:

Step 1: Find the GCF of the coefficients (numbers) within the polynomial.
Step 2: Identify the common variables contained in all terms of the polynomial.
Step 3: Multiply the result of Step 1 and 2 and contain the remaining polynomial in brackets.

Example: Factor the polynomial.

$$6x^2y^3 - 12xy^2$$

Step 1: Find the numeric GCF = 6
Step 2: Find the variable GCF = xy^2
Step 3: Find the total GCF = $6xy^2$

$$6xy^2(xy - 2)$$

Technique #1: Factoring by Grouping

This technique is used only in special cases:

1. When you are given a polynomial whose terms don't all have a single GCF. This technique is common for polynomials with greater than three terms.
2. The terms can be grouped so you can extract multiple GCFs from a few of the terms at a time to gain a common factor. (HINT: your common factor may not be a monomial.)

Example: Factor the polynomial.

$$2x^3 - 4x^2 - 3x + 6$$

$$(2x^3 - 4x^2) + (-3x + 6)$$

$$2x^2(x - 2) - 3(x - 2)$$

Notice that we have a common factor: (x-2).

This technique only works if you obtain a common factor between the two groupings.

Final Answer: $(x - 2)(2x^2 - 3)$

Technique #2: Special Factoring Patterns (SFP)

Look for handy, time-saving patterns within a polynomial. These are going to be your "bread-and-butter" for factoring special binomials. Try to become comfortable with these methods – they'll save you A LOT of time.

Definitions

Perfect Square – a single term multiplied by itself.

$$36w^4$$

$$(6w^2)(6w^2) = (6w^2)^2$$

Perfect Cube – a single term multiplied by itself twice.

$$-8y^3$$

$$(-2y)(-2y)(-2y) = (-2y)^3$$

SFP #1: The DIFFERENCE of Perfect Squares

If two squares are subtracted, $(a^2 - b^2)$, you can automatically apply the following formula to find it's factors:

$$\boxed{\mathbf{(a + b)(a - b)}}$$

Example: Factor the polynomial.

$$x^2 – 16$$

$$(x + 4)(x – 4)$$

Proof: $(x + 4)(x – 4) = x^2 – 4x + 4x – 16 = x^2 – 16$

SFP #2: The <u>DIFFERENCE</u> of Perfect Cubes

If two perfect cubes are subtracted, $(a^3 – b^3)$, they can be factored as the product of a binomial and a trinomial using the following formula:

$$\mathbf{(a - b)(a^2 + ab + b^2)}$$

Example: Factor the polynomial.

$$8x^3 – 27$$

$$a = 2x \quad b = 3$$

$$(2x – 3)((2x)^2 + (2x)(3) + (3)^2)$$

$$(2x – 3)(4x^2 + 6x + 9)$$

Proof: $8x^3 + 12x^2 + 18x – 12x^2 – 18x -27 = 8x^3 - 27$

SFP #3: The <u>SUM</u> of Perfect Cubes

If two perfect cubes are added, $(a^3 + b^3)$, they also can be factored as the product of a binomial and a trinomial using the following formula:

$$\mathbf{(a + b)(a^2 – ab + b^2)}$$

Notice that this formula differs only slightly from the previous SFP. Don’t get them confused. You’ll have to commit both of these formulas to memory for later use.

Example: Factor the polynomial.

$$(y^3 + 64)$$

$$a = y \quad b = 4$$

$$(y + 4)(y^2 – 4y + 16)$$

SFP Practice Examples:

Example 1: Factor the polynomial.

$$16x^3 + 2y^3$$

Step #1: Is there a greatest common factor (GCF)? **YES**

$$2(8x^3 + y^3)$$

Step #2: Does this expression fit a special pattern? **YES**: Sum of perfect cubes

$$(a+b)(a^2 - ab + b^2)$$

$$a = 2x \qquad b = y$$

$$(2x + y)((2x)^2 - 2xy + y^2)$$

$$(2x + y)(4x^2 - 2xy + y^2)$$

Step #3: Compose final answer. Don't forget that you took out a GCF!

$$2(2x + y)(4x^2 - 2xy + y^2)$$

Example 2: Factor the polynomial.

$$x^4 - 16$$

Step #1: Is there a greatest common factor (GCF)? **NO**
Step #2: Does this expression fit a special pattern? **YES**: Diff. of perfect squares

$$a^2 - b^2 = (a + b)(a - b)$$

$$a = x^2 \qquad b = 4$$

$$(x^2 + 4)(x^2 - 4)$$

Step #3: Compose final answer. NOT DONE YET! The second expression fits a special pattern: Diff. of perfect squares.

$$a^2 - b^2 = (a + b)(a - b)$$

$$a = x \qquad b = 2$$

$$(x^2 + 4)(x + 2)(x - 2)$$

Technique #3: Factoring Trinomials Using "The Bomb Method"

This technique can be used to solve ALL factorable trinomials! You'll find out why I call it "The Bomb Method" when you come on course. All trinomials fit into the following pattern:

$$\mathbf{ax^2 + bx + c}$$

This method requires A LOT of practice to become good at it but there are repetitive steps that you can follow to facilitate the process.

1. Factor out the GCF.
2. Find two identical numbers using the following rules:
   a. the sum of the two numbers must equal "b"; and
   b. the product of the two numbers must equal (a · c).
3. In the expression, replace "b" with the sum of the two numbers found in step #2 in brackets.
4. Distribute the variable through the brackets.
5. Use the **Factor by Grouping** technique to solve.

Sounds complicated but it isn't. Go through the following examples and it should become clear.

Example 1: Factor the polynomial.

$$6x^2 – x – 12$$

Step #1: Is there a greatest common factor (GCF)? **NO**

Step #2: This is a trinomial therefore we can use the "The Bomb Method" to factor. First, we need to find two numbers whose sum is equal to "b" and whose product is equal to (a · c).

$$b = -1$$
$$a \cdot c = 6 \cdot -12 = - 72$$

Find all of the factors for 72: (1 x 72), (2 x 36), (3 x 24), (4 x 18), (6 x 12), (8 x 9).
Remember (a · c) = -72 therefore you want one of these numbers to be negative and the other to be positive with there sum equalling -1.

$$(8 + -9) = -1$$

Step #3: Replace "b" with the sum of these two numbers.

$$6x^2 + (8 – 9)x -12$$

Step #4: Distribute the variables through the brackets.

$$6x^2 + 8x – 9x – 12$$

Step #5: Use the Factor by Grouping technique to solve.

$$2x(3x + 4) – 3(3x + 4)$$

$$(2x – 3)(3x + 4)$$

Example 2: Factor the polynomial.

$$4x^2 + 23x – 6$$

Step #1: Is there a greatest common factor (GCF)? **NO**

Step #2: This is a trinomial therefore we can use the "The Bomb Method" to factor. First, we need to find two numbers whose sum is equal to "b" and whose product is equal to (a · c).

$$b = 23$$
$$a \cdot c = 4 \cdot -6 = -24$$

Find all of the factors for 24: (1 x 24), (2 x 12), (3 x 8) & (4 x 6)
Remember (a · c) = -24 therefore you want one of these numbers to be negative and the other to be positive with there sum equalling -23.

$$(-1 + 24) = 23$$

Step #3: Replace "b" with the sum of these two numbers.

$$4x^2 + (24 – 1)x – 6$$

Step #4: Distribute the variables through the brackets.

$$4x^2 + 24x – x – 6$$

Step #5: Use the Factor by Grouping technique to solve.

$$4x(x + 6) – 1(x + 6)$$

$$(4x – 1)(x + 6)$$

Technique #4: Special Factoring Patterns (SFP) for Trinomials

To complicate things there are special factoring patterns for trinomials. "The Bomb Method" will work for all trinomials except those with more than one variable. These more complicated polynomials may have a factoring pattern that can be followed. We will only be looking at two of these cases for the purpose of this course: **Perfect Square Trinomials.**

Perfect Square Trinomials are products of the algebraic expressions $(a + b)^2$ and $(a – b)^2$. Distributing the variables we have the following patterns:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a – b)^2 = a^2 – 2ab + b^2$$

For a trinomial to be a perfect square trinomial the following conditions must be met:

1. The first term, "a", is a perfect square.
2. The last term, "b", is a perfect square.
3. The middle term is either 2 or -2 times the product of the square root of the first term and the square root of the last term (a · b).

Example 1: Factor the polynomial.

$$4x^2 + 4xy + y^2$$

Step #1: Is there a greatest common factor (GCF)? **NO**

Step #2: This is a trinomial with two variables (x and y) therefore we cannot use the "The Bomb Method" to factor. We need to identify is the first term is a perfect square: **YES**

$$\sqrt{4x^2} = 2x$$

Step #3: Is the last term a perfect square? **YES**

$$\sqrt{y^2} = y$$

Step #4: Is the middle term 2 times or -2 times the product of 2x and y? **YES**

$$2 \cdot 2x \cdot y = 4xy$$

Step #5: The product is a positive term therefore it will fit the following pattern:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(2x + y)^2$$

<u>Technique #5: Factoring Higher Order Polynomials</u>

So far, we've only introduced factoring methods for quadratic equations and a few cubic equations – next on the list is looking at factoring polynomials with higher powers. The techniques you use to solve 3rd-degree polynomials are the same used for 4th-, 5th and 50th-degree equations. There are no new skills that need to be learned as long as you are comfortable with the long division of polynomials.

The first step is to demonstrate that a specific linear polynomial is a factor of the higher order polynomial using long division. How do you decide which linear polynomial to use? Use the **rational root test**.

Rational Root Test

This test uses the leading coefficient and constant of a higher order polynomial to generate a list of all possible solutions for the unknown variable. Unfortunately some trial and error is necessary until you can identify the proper solution. This method works for all higher degree polynomials.

Step #1: Identify all the factors of the leading coefficient (L).

Step #2: Identify all the factors of the constant (C).

Step #3: List all possible combinations of L and C in the form: $\pm \frac{C}{L}$

Step #4: Try each of the potential solutions from the list – applying them to the polynomial using long division.

When testing potential factors using long division there should be NO REMAINDER. Continue to factor the polynomial using long division and the potential roots from the rational root test until you have completely factored the higher order polynomial.

Example 1: Factor the higher order equation.

$$2x^3 - 9x^2 - 8x + 15 = 0$$

Step #1: Identify all the factors of the leading coefficient (L).

Leading Coefficient (L) = 2
Factors: 1, 2

Step #2: Identify all the factors of the constant (C).

Constant (C) = 15
Factors: 1, 3, 5, 15

Step #3: List all possible combinations of L and C in the form: $\pm \frac{C}{L}$

Possible roots: $\pm 1,\ \pm\frac{1}{2},\ \pm 15,\ \pm\frac{15}{2},\ \pm 3,\ \pm\frac{3}{2},\ \pm 5,\ \pm\frac{5}{2}$

Step #4: Try each of the potential solutions from the list – applying them to the polynomial using long division.

HINT: Always start testing the potential solutions that are whole numbers (much easier to use long division). Therefore: $\pm 1,\ \pm 3,\ \pm 5,\ \pm 15$. You will only need to progress to the fractions if one of these whole numbers does not pan out into a solution.

Try x = -1 first:

$$x = -1$$
$$x + 1 = 0$$

$$\begin{array}{r} 2x^2 - 11x + 3 \\ x+1\overline{\big)2x^3 - 9x^2 - 8x + 15} \\ -\underline{(2x^3 + 2x^2)} \Downarrow \\ -11x^2 - 8x \\ \underline{-(-11x^2 - 11x)} \Downarrow \\ 3x + 15 \\ \underline{-(3x + 3)} \\ 12 \end{array}$$

$$REMAINDER \therefore NO!$$

Try x = 1 next:

$$x = 1$$
$$x - 1 = 0$$

$$\begin{array}{r} 2x^2 - 7x - 15 \\ x-1\overline{\big)2x^3 - 9x^2 - 8x + 15} \\ -\underline{(2x^3 - 2x^2)} \Downarrow \\ -7x^2 - 8x \\ -\underline{(-7x^2 + 7x)} \Downarrow \\ -15x + 15 \\ -\underline{(-15x + 15)} \\ 0 \\ \therefore YES! \end{array}$$

The final step is to factor the solution to the long division, i.e.: $2x^2 - 7x - 15$. This is a trinomial (degree = 2) therefore you can use the simpler Bomb Method to solve.

Step #1: Is there a greatest common factor (GCF)? **NO**

Step #2: This is a trinomial therefore we can use the "The Bomb Method" to factor. First, we need to find two numbers whose sum is equal to "b" and whose product is equal to (a · c).

b = -7

a · c = 2 · -15 = - 30

Find all of the factors for 30: (1 x 30), (2 x 15), (3 x 10), (5 x 6).
Remember (a · c) = -30 therefore you want one of these numbers to be negative and the other to be positive with there sum equalling -7.

(3 + -10) = -7

Step #3: Replace "b" with the sum of these two numbers.

$2x^2 + (3-10)x - 15$

Step #4: Distribute the variables through the brackets.

$2x^2 + 3x – 10x – 15$

Step #5: Use the Factor by Grouping technique to solve.

x(2x + 3) – 5(2x + 3)

(x-5)(2x+3)

FINAL SOLUTION: $2x^3 - 9x^2 - 8x + 15 = (x-1)(x-5)(2x+3)$

CHAPTER 8 - AMS 101- Applied Mathematics Formula Sheet

Special Factoring Patterns

Difference of Perfect Squares ($a^2 - b^2$) $(a+b)(a-b)$

Difference of Perfect Cubes ($a^3 - b^3$) $(a-b)(a^2+ab+b^2)$

Sum of Perfect Cubes ($a^3 + b^3$) $(a+b)(a^2-ab+b^2)$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solving Higher Order Equations

Rational Root Test: L = leading coefficient factors
C = constant factors
$\pm \frac{C}{L}$

Measurement of Angles

360º = 6400 mils = 2π rads

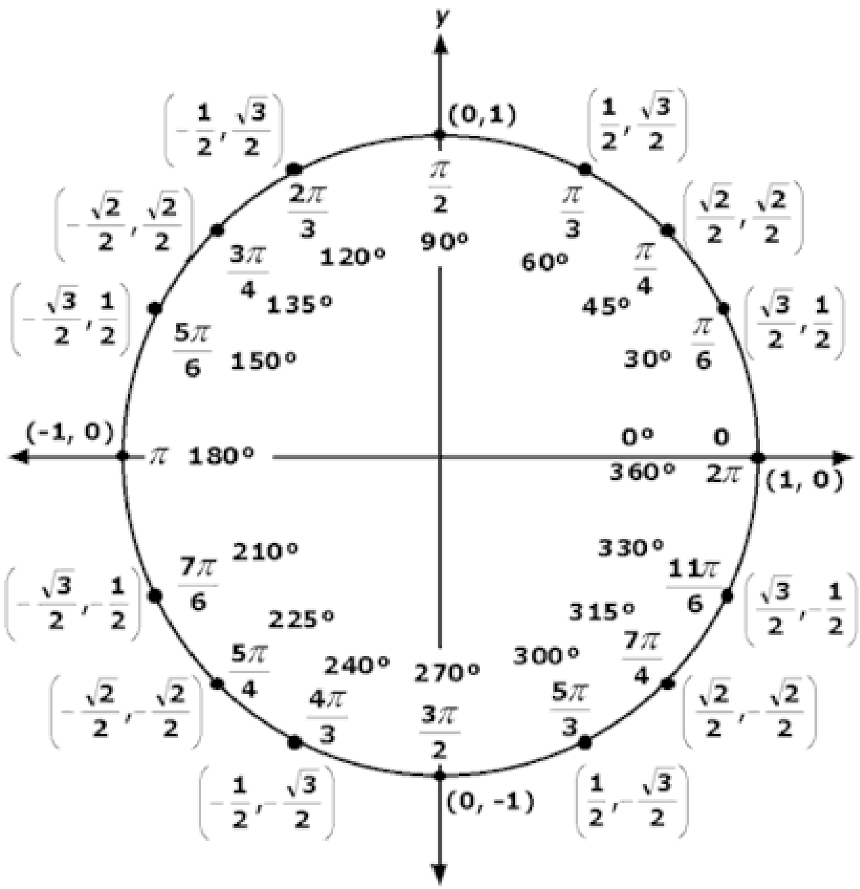
Even/Odd Trigonometry Functions

$$\sin(-x) = -\sin(x)$$
$$\cos(-x) = \cos(x)$$
$$\tan(-x) = -\tan(x)$$

Trigonometric Functions

	Domain (General Function)	Range (General Function)	Period (T)
$y = a\sin(bx - c) + d$	$x =$ any angle	$-1 \le \sin x \le 1$	$\frac{2\pi}{b}$
$y = a\cos(bx - c) + d$	$x =$ any angle	$-1 \le \cos x \le 1$	$\frac{2\pi}{b}$
$y = a\tan(bx - c) + d$	$x \ne (n + \frac{1}{2})\pi$ $\mathrm{n} = 0, \pm 1, \pm 2 \ldots$	$-\infty \le \tan x \le \infty$	$\frac{\pi}{b}$

Unit Circle



Pythagorean Theorum

$$H^2 = A^2 + B^2$$

Law of Sines

* Use for ambiguous case determinations.

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Complex Numbers

$$i = \sqrt{-1}$$

Graphing

Linear Equation: $y = mx + b$ $\quad m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$

Circle Equation: $(x-h)^2 + (y-k)^2 = r^2$

Oval Equation: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Logarithm Rules

$\log_b(b^x) = x$, when $b \neq 1$

$c = \log_b(a) \quad \rightarrow \quad a = b^c$

$\log_b(x^y) = y \log_b(x)$

*** When no logarithmic base (*b*) is identified assume *b* = 10.**

$\ln(x^y) = y \ln(x)$

$\ln(e^x) = x$